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Fifth Quarterly Report

on

PROJECT STAR DUST

by

James P. Friend Herbert W. Fooly

Prepared for the Defense Atomic Support Agency

August 1, 1962

ISOTOPES, INC.

DASA 1305

Fifth Quarterly Report

on

PROJECT STAR DUST

by

James P. Friend and Herbert W. Feely

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ABSTRACT

Two preliminary versions of the Star Dust model of atmospheric mixing and transfer have been run on a computer and have been checked against HASP tungsten-185 data. The results obtained have tended to substantiate the validity of the basic design of the model and have indicated some modifications which are needed in it.

Most results are now available from radiochemical analysis of Star Dust samples collected during early 1962. These include preliminary measurements of several products of neutron activation in the debris from 1961 Soviet tests.

The total burden of strontium-90 in the stratosphere of the Northern Hemisphere during January - April 1962 was approximately 1.6 or 1.7 megacuries, about 1.3 or 1.4 megacuries of which had been produced by the 1961 Soviet tests. The total burden of strontium-90 in the entire stratosphere was probably about 2 megacuries at that time.

There is evidence that the layer of air in the northern polar stratosphere which contained the highest concentrations of strontium-90 from pre-1961 tests had subsided during the winter of 1961 - 1962 from an altitude of 70,000 feet or higher in mid-1961 to about 60,000 feet by early 1962. The highest concentrations of debris from the 1961 Soviet tests continued to be found at 50,000 feet in the northern polar stratosphere throughout early 1962, but vertical mixing gradually increased the low concentrations of debris at 65,000 and 70,000 feet. The influx of antimony-124 into this highest sampled layer during early 1962 indicates that debris from above 70,000 feet must have contributed, in part at least, to this increase. Less than 10 percent of the debris from the 1961 Soviet tests had mixed into the tropical stratosphere by May 1962.

The mean altitude of stabilization of debris from the 1961 Soviet tests was probably at least 10,000 feet above that of debris from the October 1958 Soviet tests. As a result the debris from the 1961 tests will exhibit a longer stratospheric residence time, with perhaps half of it being detained in the stratosphere for more than a year before falling out.

Changes which occurred in concentrations of debris near the tropopause during early 1962 suggest that both the rising of the polar tropopause during the spring months and horizontal mixing through the tropopause gap region (or through the sloping polar tropopause) are important in contributing to the increase in fallout rate at the surface of the earth during the spring season.

PURPOSE AND METHOD

Project Star Dust has been undertaken to prepare a mathematical model of atmospheric mixing and transport which can be used to predict stratospheric hold-up and ultimate patterns of deposition on the surface of the earth of radioactive debris injected into the stratosphere by nuclear weapons tests, by burn-up on re-entry of nuclear power packs for earth satellites, or by other causes. Information on several atmospheric "tracers" is being used to guide the selection of values for the various meteorological parameters included in the model. Among these tracers are ozone, water vapor and such radionuclides as strontium-90, tungsten-185, rhodium-102 and beryllium-7, which have been introduced into the stratosphere by nuclear weapons tests or by natural atmospheric processes.

Data from the High Altitude Sampling Program and from other studies of atmospheric radioactivity are being used to determine atmospheric distributions of the radioactive tracers, but additional measurements of this radioactivity are also being made during Project Star Dust. These measurements have already provided a good deal of valuable information on rate of fallout of debris from nuclear weapon tests performed before 1959 and on the original distribution and subsequent movement of debris from the 1961 Soviet tests. Similar information concerning debris from the 1962 United States tests is now becoming available.

RECENT PROGRESS

Work on the Star Dust model of stratospheric mixing and transfer was begun under the supervision of Dr. Edwin L. Fisher. Since the death of Dr. Fisher in June 1962 this work has been supervised by Dr. Benjamin Davidson of New York University. Thus far a diffusion model has been programmed and tested on a CDC 1604 computer under two sets of conditions, one without and one with a tropopause but with no tropospheric sink. Next a tropospheric sink will be added. Data on stratospheric distributions of tungsten-185 following the 1958 Hardtack test series are being used to test the acceptability of the results of these trials.

The Star Dust sampling program proceeded well during May, June and July 1962, though no orbit missions were flown at Laughlin or Eielson during July. Some samples were received from Howard Air Force Base, the Canal Zone, during June and July, however, adding to our information on the stratospheric distribution of debris from the 1962 United States tests at Christmas Island. The first interception of debris from these tests by Star Dust aircraft occurred on 8 May 1962. Subsequent interceptions of this debris were made during May, June and July by aircraft stationed at Laughlin and Howard, and at least once, by aircraft stationed at Eielson.

Results from the radiochemical analysis of Star Dust samples have been submitted to Headquarters DASA in monthly letter reports beginning with a report

dated 31 March 1962. Initial results from the analysis of some Star Dust samples for several products of neutron activation, evidently injected into the stratosphere by the 1961 Soviet weapon tests, are presented in this report.

STRATOSPHERIC CONCENTRATIONS OF NUCLEAR DEBRIS DURING EARLY 1962

A large proportion of nuclear debris injected into the stratosphere of the northern hemisphere by the Soviet weapon tests of late 1961, together with the residue of debris from tests performed before 1959, was still present in the stratosphere during the early months of 1962. In late April 1962 the United States resumed weapon tests in the atmosphere, with the resultant addition of significant amounts of fresh debris to the stratospheric reservoir.

The Distribution of Nuclear Debris in the Star Dust Corridor.

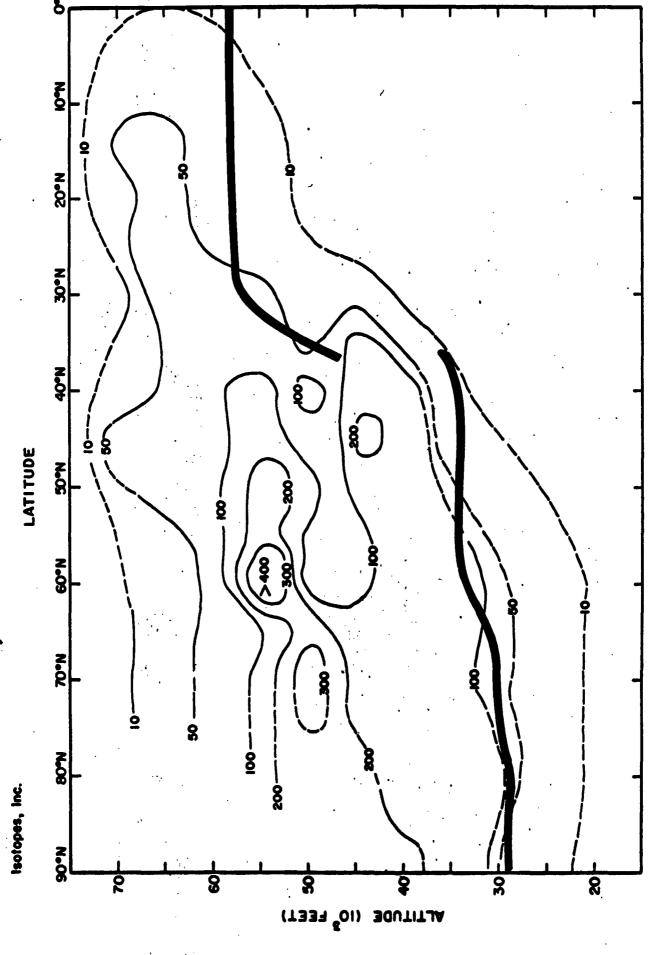
Measurements of the total beta activity of Star Dust filter samples have been used to calculate the mean distribution of nuclear debris in the Star Dust sampling corridor during March, April, May and June 1962 (Figures 1 through 4). The mean position of the tropopause during each of these months is indicated by heavy lines in these figures (and in subsequent cross-sections). Interpolated and extrapolated isopleths of beta activity are dashed in regions where data are lacking.

During March and April 1962 the highest stratospheric concentrations were encountered in the polar stratosphere at 50,000 to 60,000 feet. Activities decreased sharply with altitude above 60,000 feet in the polar stratosphere.

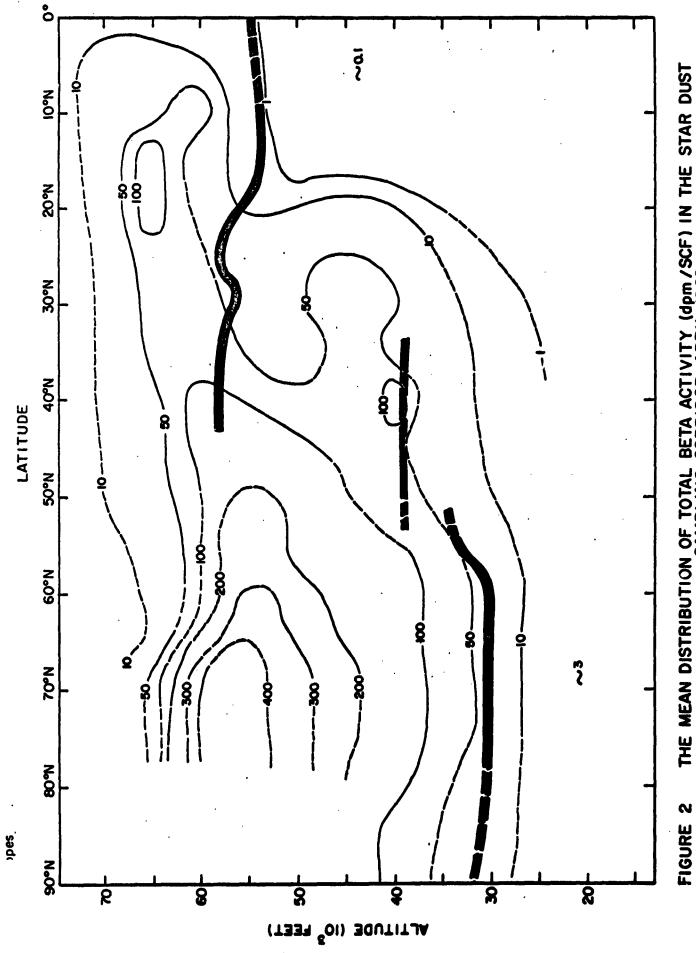
As in earlier months there was evidence that little Soviet debris had mixed as far south as the equator, but that the debris which did move southward rose

to higher altitudes as it entered the tropical stratosphere. (See the Third and Fourth Quarterly Reports on Project Star Dust 1, 2).

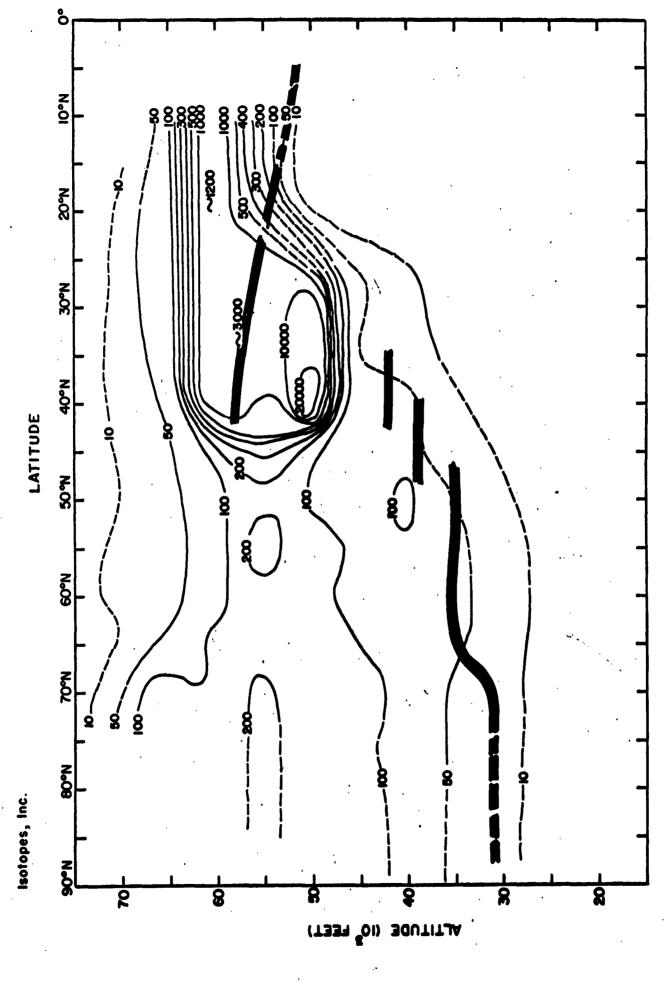
During May and June 1962 debris from the 1962 United States tests at Christmas Island was encountered in the tropical stratosphere and at intermediate latitudes. By June small quantities of this debris had reached high northern latitudes. During June also a layer of air containing high concentrations of Soviet debris was encountered at high altitudes in the northern polar stratosphere. A layer of air containing much lower concentrations was sandwiched between it and the mass of debris in the layers at 45,000 to 55,000 feet. Each of these layers sloped downward toward the pole. Possibly this high altitude layer containing Soviet debris had been present above 70,000 feet in the polar regions throughout early 1962 but had subsided to altitudes where it could be sampled by Star Dust aircraft only during June. It is probably more likely that this layer had been present at 60,000 to 70,000 feet during early 1962 and had indeed been sampled occasionally by Star Dust aircraft. In this latter case it would have to be assumed that the Soviet debris was not yet distributed uniformly about the polar axis of the zonal flow of the atmosphere, so that successive sampling missions in the same region could encounter quite different concentrations of debris. Of course a good deal of evidence does support this assumption. (As further evidence that the distribution of activity in the northern polar stratosphere found during June 1962 was probably not the result of extensive vertical movement of debris, it may be mentioned that the high altitude layer of high activities was not encountered by Star Dust sampling during July 1962).



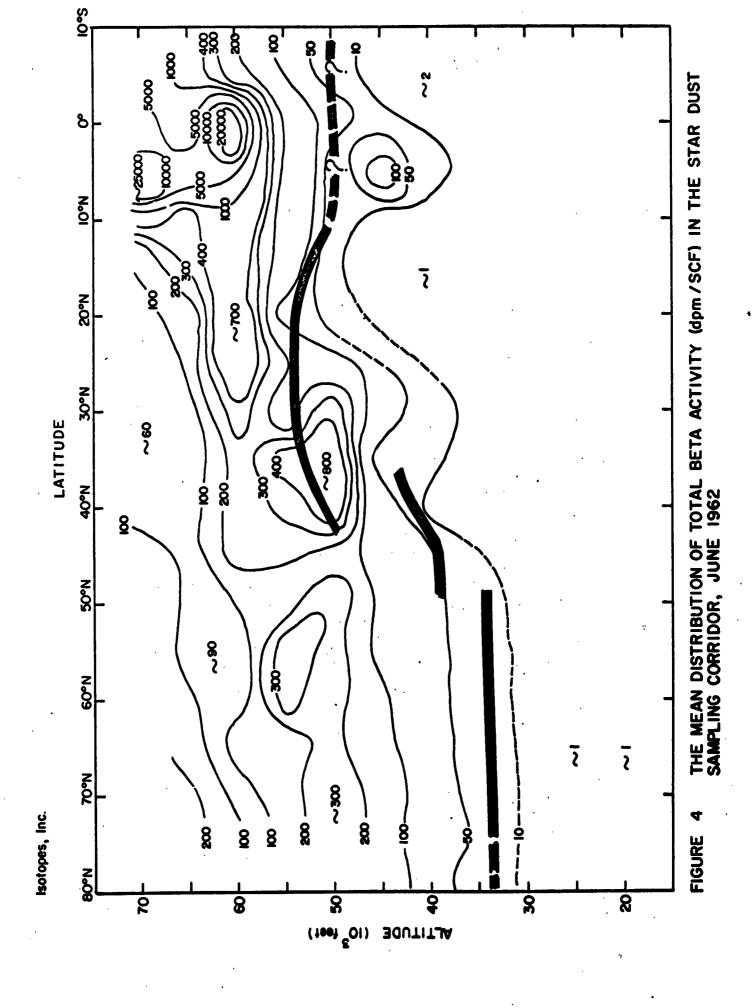
THE MEAN DISTRIBUTION OF TOTAL BETA ACTIVITY (dpm/SCF) IN THE STAR DUST SAMPLING CORRIDOR, MARCH 1962 FIGURE



THE MEAN DISTRIBUTION OF TOTAL BETA ACTIVITY (dpm/SCF) IN THE STAR DUST SAMPLING CORRIDOR, APRIL 1962



THE MEAN DISTRIBUTION OF TOTAL BETA ACTIVITY (dpm/SCF) IN THE STAR DUST SAMPLING CORRIDOR MAY 1962 FIGURE 3



The distribution of strontium-90 in the Star Dust sampling corridor during late 1961 and early 1962 is shown in Figures 5 through 8. Since January 1962, when Star Dust sampling of the northern polar stratosphere was begun, the highest concentrations of strontium-90 from the 1961 Soviet tests have been found in the polar stratosphere at 50,000 to 60,000 feet altitude. The concentrations of this debris in the northern tropical stratosphere decreased from late 1961 into January - February 1962 and perhaps into March - April, but any possible subsequent changes were obscured by the high strontium-90 concentrations produced in this area by the 1962 U.S. tests.

The Stratospheric Burden of Strontium-90 During Early 1962.

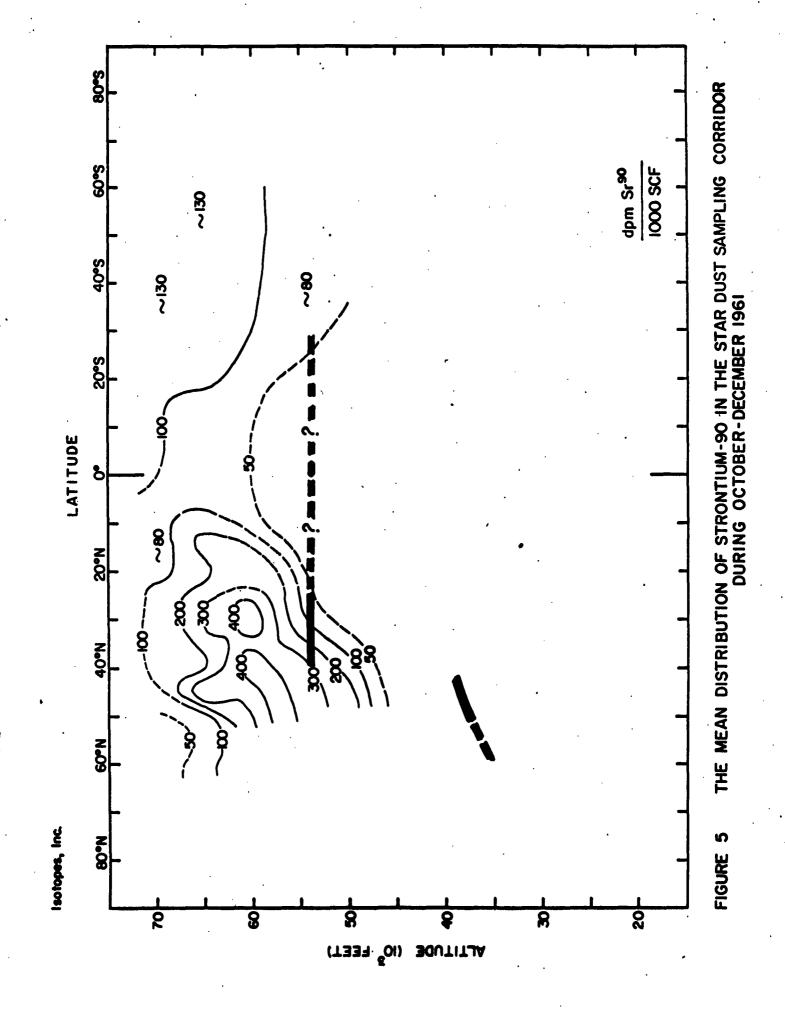
The Star Dust measurements of stratospheric concentrations of nuclear debris may be used as a basis for the calculation of stratospheric burdens of strontium-90 resulting from specific series of weapon tests. Thus, using data now in hand we may estimate the burden of strontium-90 produced by the 1961 Soviet test series. In the near future, when more data are available, it should also be possible to undertake the calculation of the stratospheric burden of strontium-90 resulting from the 1962 United States test series.

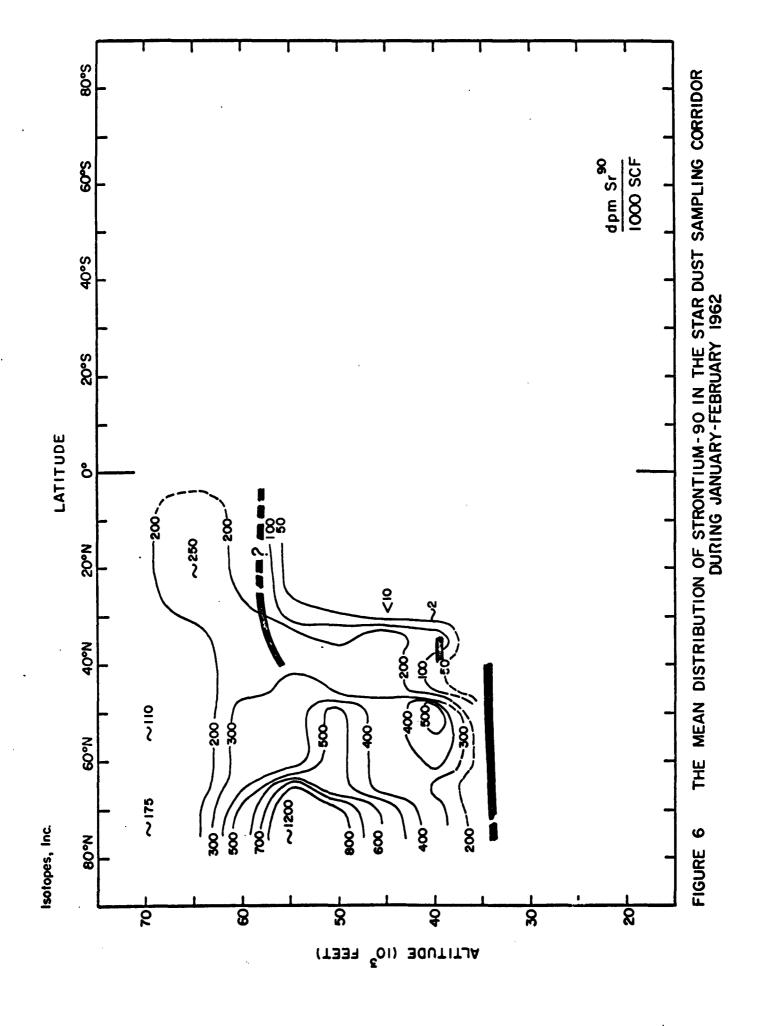
Here we will estimate the stratospheric burden during January to

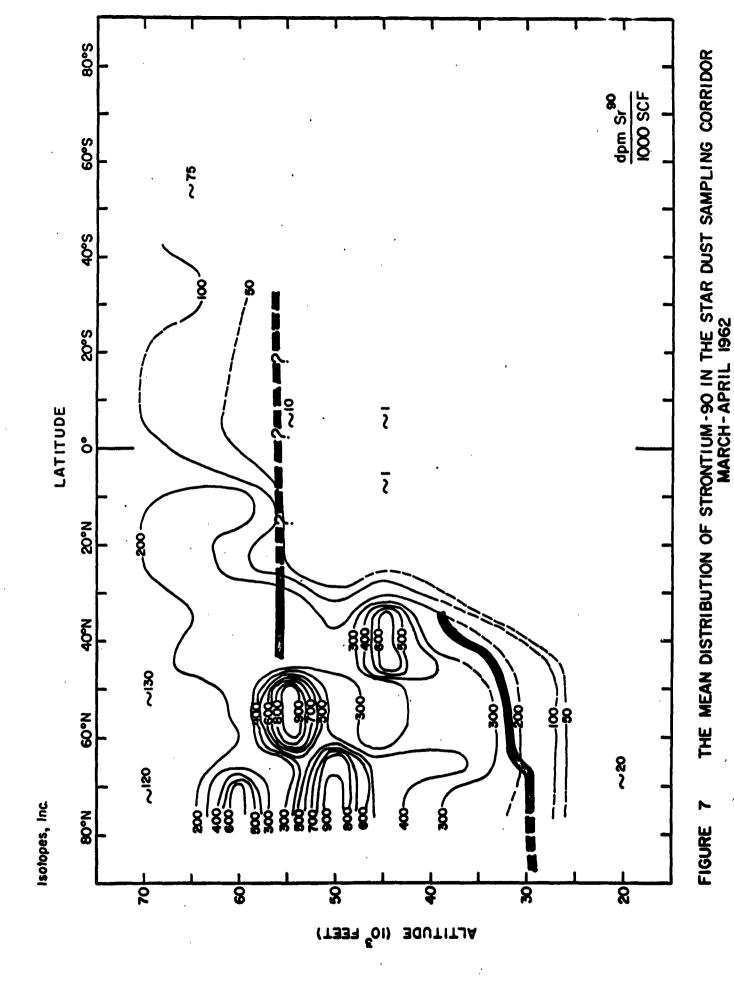
April 1962 of strontium-90 from the 1961 Soviet tests. Data for samples collected

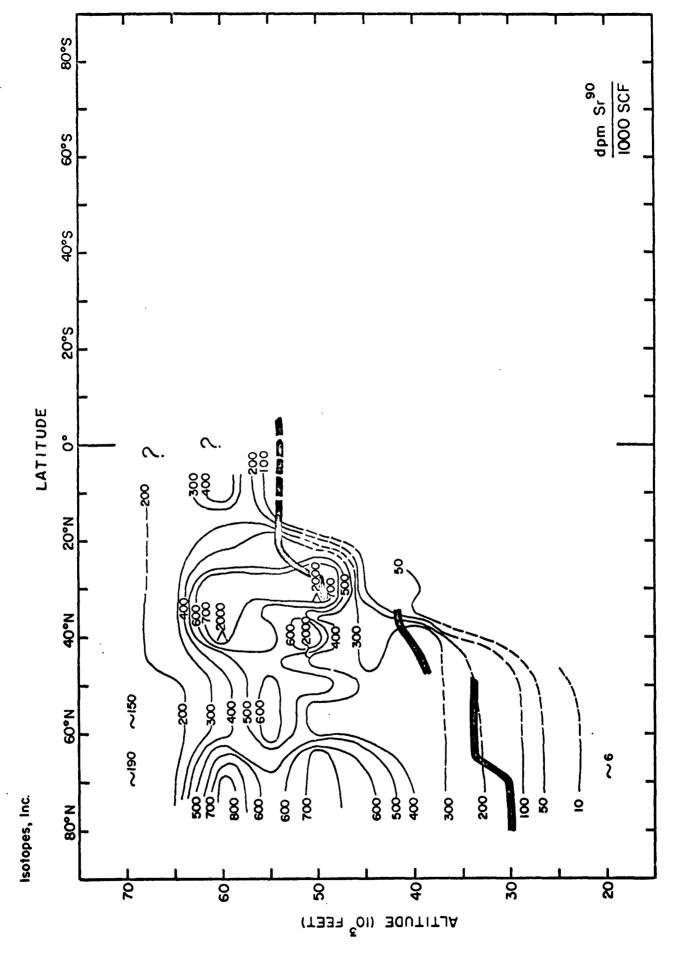
before January 1962 are excluded because Star Dust sampling did not extend

northward beyond 49°N until January, and data for samples collected during









THE MEAN DISTRIBUTION OF STRONTIUM-90 IN THE STAR DUST SAMPLING CORRIDOR DURING MAY-JUNE 1962 FIGURE 8

and after May 1962 are excluded because of the complications introduced by the presence in such samples of debris from the 1962 United States tests. There are, of course, several limitations on the accuracy of this calculation which should be borne in mind. For example there is the possibility that samples collected in the Star Dust corridor during the four month period in question were not truly representative of the mean concentrations of debris in the entire stratosphere. Ihe rather pronounced variations in concentration observed from mission to mission and from month to month suggest that this factor could easily introduce an uncertainty of + 20 percent into the calculation. The undoubted presence at altitudes above 70,000 feet of some of the debris from the high yield Soviet tests of 23 or 30 October 1961 could result in our underestimating the total burden, though an attempt is made to include this debris in the calculation. The continual loss of debris from the stratosphere during the four months represented by this inventory calculation naturally complicates the assessment of the significance of the result. Finally there is the necessity of distinguishing between strontium-90 from the Soviet tests and strontium-90 from earlier tests.

In the past² we have corrected the strontium-90 concentrations observed during early 1962 for the presence of debris from pre-1961 sources by subtracting from them the concentrations of strontium-90 observed in the stratosphere during June to September 1961. This procedure involves the assumption that no change occurred in the stratospheric distribution of old debris in the Northern Hemisphere

during the winter of 1961 - 1962. Such an assumption may be of questionable validity in view of the profound changes in the distribution of debris which have been observed to occur during previous winters. Nevertheless it has been used in the past because of the simplicity of the calculation involved and because of the feeling that the error introduced by this assumption was not large. We will use this procedure again in this report to calculate the stratospheric burden during January to April 1962 of strontium-90 from the 1961 Soviet tests. But in addition we will apply a second procedure, one involving the use of Sr^{89}/Sr^{90} ratios in the samples for distinguishing debris from the 1961 tests, to perform an alternate calculation.

In Figure 9 is shown the mean stratospheric distribution of strontium-90 in the Star Dust sampling corridor during January - April 1962. We will assume that the distribution in this quasi-meridional plane is representative of that in all meridional planes through the stratosphere of the Northern Hemisphere.

In Figure 10 is shown the approximate mean distribution of strontium-90 in the Star Dust corridor during June - September 1961. Subtraction of the concentrations represented by Figure 10 from those represented by Figure 9 gives us Figure 12A as the apparent distribution of new Soviet debris in the stratosphere during early 1962. If, instead of this subtraction, we use the Sr⁸⁹/Sr⁹⁰ ratios observed in the debris present in the stratosphere during early 1962 we obtain a somewhat different result. We have arbitrarily assumed a mean shot date for the 1961 Soviet test series of 15 October 1961. We have then assumed

that on that date 38.9 was the initial value of the Sr⁸⁹/Sr⁹⁰ activity ratio³. With these assumptions we have calculated the contribution of the 1961 Soviet series and of earlier tests to the 1962 burden. The apparent distribution of pre-1961 debris thus calculated is shown in Figure 11 while the apparent distribution of 1961 Soviet debris is shown in Figure 12B. (The significance of the differences between Figures 10 and 11 will be discussed in the next section.) It is evident that there is not very much difference between Figures 12A and 12B, though the concentrations indicated at 70,000 feet at very high northern latitudes are higher in Figure 12B and the concentrations indicated at 40,000 to 50,000 between 30 °N and 60 °N are lower. Figure 12B gives more indication than does Figure 12A that significant quantities of Soviet debris may have been injected above 70,000 feet in late 1961. Considering the assumptions on which these two figures are based, Figure 12B seems the more reliable representation of the Soviet debris.

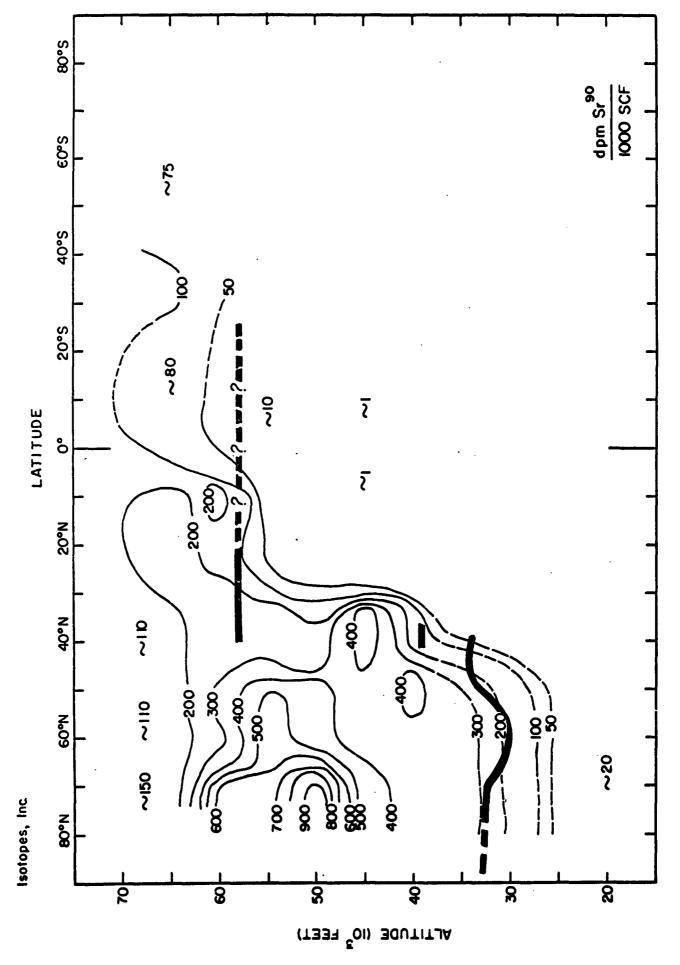
The distributions portrayed in Figures 12A and 12B have been extrapolated to the North Pole and to the top of the atmosphere and a mean strontium-90 concentration has been estimated for each stratospheric region in the Northern Hemisphere. In extrapolating data in Figure 12B to the top of the atmosphere a layer of air containing over 300 dpm Sr⁹⁰/1000 SCF has been assumed to exist between 80,000 and 90,000 feet and between 40°N and 90°N. The data in Tables 1 and 2 correspond to the distributions in Figures 12A and 12B respectively.

The stratospheric burden of strontium-90 from the 1961 Soviet tests has been calculated to be 1.36 megacuries in Table 1 and 1.31 megacuries in Table 2. A reasonable value appears to be 1.3 + 0.3 megacuries. Both Tables 1 and 2 indicate that 91 percent of this debris, about 1.2 megacuries, was still north of 30°N.

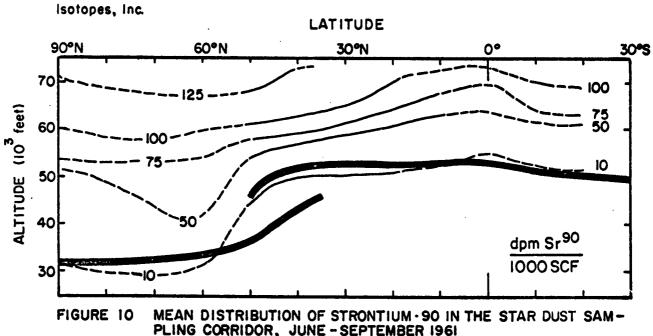
A calculation of the stratospheric burden of strontium-90 in the Northern Hemisphere during June - September 1961, using the distribution shown in Figure 10, gave 0.35 megacurie. From Figure 11 we might calculate that 0.34 megacurie of old strontium-90 was still present in the stratosphere of the Northern Memisphere in early 1962. This calculation is given in Table 3. A drop in the burden of only 0.01 megacurie during the interval August 1961 - March 1962 seems rather low. Since the extrapolations of Star Dust data to high altitudes and high latitudes required for the calculation of the June - September 1961 burden introduced quite significant uncertainties into the calculations, it is quite possible that the burden during that interval has been slightly underestimated. Evidently the total strontium-90 burden in the stratosphere of the Northern Hemisphere during early 1962 was about 1.6 to 1.7 megacuries, and the burden in the entire stratosphere was probably about 2 megacuries.

It may be of interest to compare the distribution of nuclear debris in the stratosphere in early 1962 with that which was found in early 1959, following the series of United States weapon tests in mid-1958, the United Kingdom tests in September 1958, and the intensive series of Soviet tests in October 1958.

In Figure 13 is shown the mean distribution of strontium-90 in the HASP sampling corridor during January - April 1959. In contrast to the distributions shown in Figure 12 it can be seen that in early 1959 the highest concentrations of strontium-90 encountered were in the tropical stratosphere and at the highest altitudes sampled in the northern polar stratosphere. Actually this debris was virtually all derived from the tropical injections by United States and United Kingdom tests. Very little debris from the 1958 Soviet tests was encountered, apparently because it was almost completely limited to the layers of polar air below 50,000 feet, a region not sampled by routine HASP missions at that time. The only samples of this Soviet debris obtained during early 1959 came from a mission flown by a B-52 aircraft at altitudes of about 40,000 feet in April 1959. The position of this debris, so close to the polar tropopause, doubtless explains its almost complete loss from the stratosphere during the Spring of 1959. The presence of the Soviet debris at much higher altitudes in 1962 than in 1959 is almost certainly attributable to the higher average yield of the weapons tested in the more recent series. One result of this difference is that much of the debris from the 1961 tests will remain in the stratosphere for more than a year.



THE MEAN DISTRIBUTION OF STRONTIUM-90 IN THE STRATOSPHERE DURING JANUARY-APRIL 1962 ത FIGURE



PLING CORRIDOR, JUNE - SEPTEMBER 1961

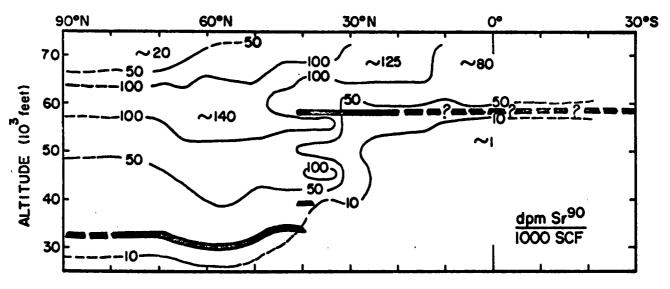


FIGURE 11 MEAN DISTRIBUTION OF STRONTIUM-90 FROM PRE-1961 TESTS STILL PRESENT IN THE STAR DUST SAMPLING CORRIDOR DURING JANUARY-APRIL 1962, CALCULATED FROM Sr89 /Sr90



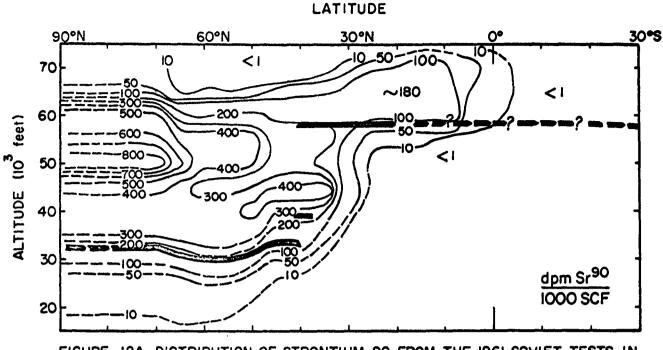


FIGURE 12A DISTRIBUTION OF STRONTIUM-90 FROM THE 1961 SOVIET TESTS IN THE STAR DUST CORRIDOR DURING JANUARY-APRIL 1962, USING JUNE-SEPTEMBER 1961 DISTRIBUTION TO CORRECT FOR PRE-1961 STRONTIUM-90

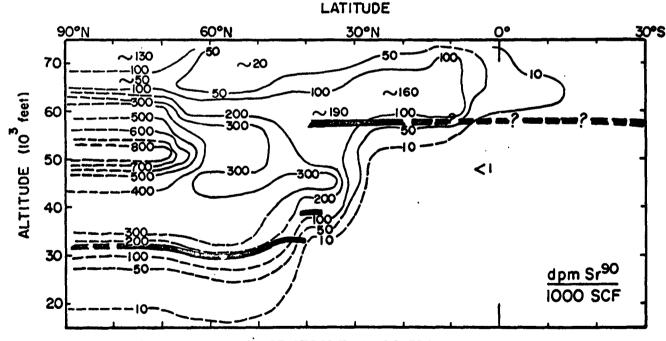


FIGURE 128 DISTRIBUTION OF STRONTIUM-90 FROM THE 1961 SOVIET TESTS IN THE STAR DUST CORRIDOR DURING JANUARY-APRIL 1962, USING Sr89/Sr90 TO CORRECT FOR PRE-1961 STRONTIUM-90

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Calculation of stratospheric burden during January - April 1962 of strontium-90 from the 1961 Soviet test series, based on the stratospheric distribution shown in Figure 12A Table 1.

								İ			
Pressure	08	02	09	50				10			
(mp)	N. 06	%0 %N	N. 02	N. 09	20 N	40 °N	30 °N	20 °N	N. 01	S 01 1	S. 02
0-40	Ŋ	ß	7	ı				30			
40-80	200	200	100	110				140			
80-120	100	650	525	400				15			
120-160	009	009	400	320				•			
160-200	370	370	330	350				•			
200-240	320	320	340	350				•			
240-280	245	245	275	300				•			
280-320	130	130	150	175				•			
320-360	20	70	80	96				•			
360-400	40	40	45	20				•			
400-440	52	52	52	30				•			
440-480	20	70	20	52				•			
480-520	10	10	12	15				•			
520-560	2	2	5	∞				•			
260-600	-	T	-	-				•			
C (qpm											
Z \ 1000SCF		2691	2310	2224	1740	1053	526	185	85	13	•
Area $(10^8 \text{mi} \stackrel{?}{3} 0.015)$			0.074		0, 123 214, 0	0. 143 150. 6	0.158 40.4	0, 168	0. 174 14.8	0.174	0, 168 0, 0

(1008.7) (13.48×10^{-4}) = 1.36 megacuries

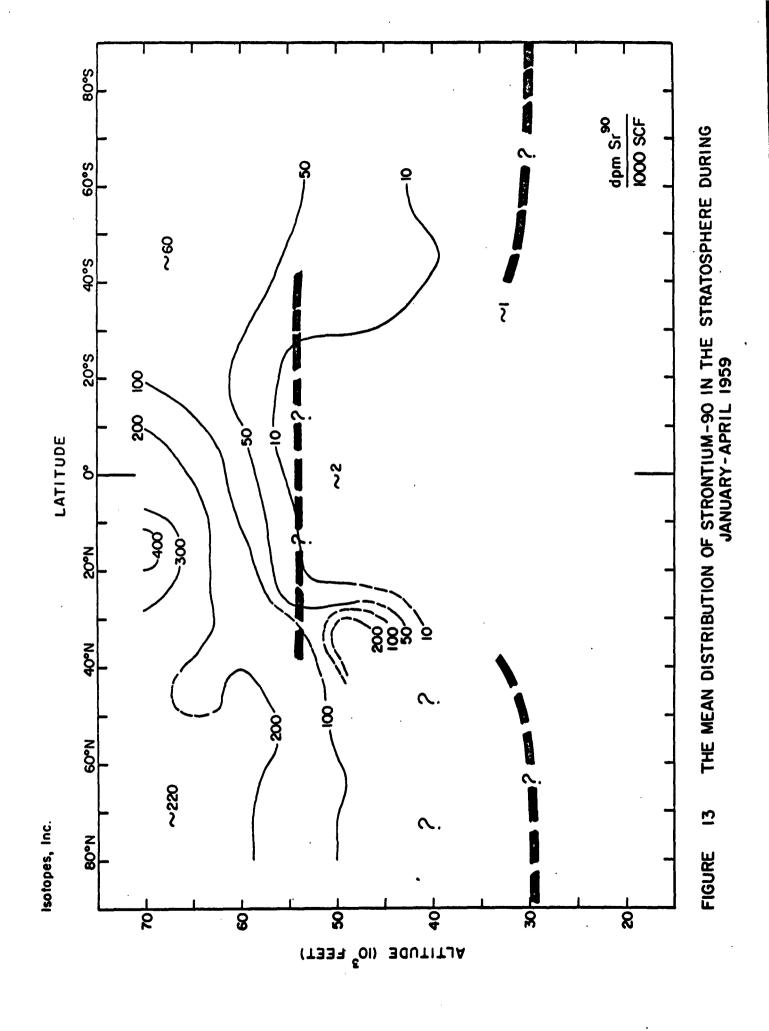
Calculation of stratospheric burden during January - April 1962 of strontium-90 from the 1961 Soviet test series, based on the stratospheric distribution shown in Figure 12B. Table 2.

Pressure (mb)	80°- 80°N	70°- 80°N	N. 02	20 ° 09	40°-	30°-	20°-	10°- 20°N	0°- 10°N	0 10 .5	10°- 20°S
0-40	230	230		160	160			15	20		7
40-80	240	240		96	100			120	20		Ŋ
80-120	200	650		350	220			20	9		1
120-160	550	530		300	300			-	7		•
160-200	380	380		310	290			•	1	ı	ı
200-240	330	330		330	200			1	1		•
240-280	250	250		300	165			•	•		1
280-320	140	140		190	09			•	ı	ı	•
320-360	75	75		90	40			1	ı	•	1
360-400	30	33		55	07			•	1	t	•
400-440	52	52		52	10		•	•	•	•	•
440-480	15	15		15	10		•	•	•	•	ı
480-520	10	10		12	2		ı	•	1	t	•
520-560	5	2		œ	7		1	1	1	1	ı
260-600	1	7		-	•		1	ı	•	1	1
\											
Z/ 1000 SCF /	2981	2911	2429	2236	1582	197	232	156	26	16	9
Area(108mi2) 0.015		0,045	0.074	0, 100	0.123	0, 143	0,158	0, 168	0, 174	0.174	0, 168
$(\sum)^{(Area)}$	44.7	44.7 131.0	179.7	223.6	194.6	114.0	36.7	26.2	16.9	2.8	1.0

 $(971.2) (13.48 \times 10^{-4}) = 1.31 \text{ megacuries}$

Table 3. Calculation of stratospheric burden during January - April 1962 of strontium-90 from weapons tests before 1961, based on the stratospheric distribution shown in Figure 11.

Pressure (mb)	80 °- 90 °N	70°- 80°N	60°- 70°N	50°-	40 °- 50 °N	30 °- 40 °N	20 °- 30 °N	10 °- 20 °N	0°- 10°N
0 - 40	2	2	2	10	10	20	35	40	40
40 - 80	95	95	100	100	90	85	90	85	65
80-120	75	80	105	110	75	60	15	10	5
120-160	50	50	60	60	70	75	5	1	1
160-200	25	25	50	50	50	4 0	8	1	1
200-240	20	20	30	30	25	8	2	1	-
240 - 280	20	20	25	20	15	5	-	-	-
280 - 320	15	15	20	20	10	3	-	-	-
320 - 360	10	10	11	12	9	2	-	-	-
360 - 400	9	9	10	10	5	1	-	-	-
400-440	7	7	8	8	4	-	-	•	•
440 - 480	5	5	5	5	2	-	-	-	-
480-520	3	3	3	3	1	-	-	-	-
520-560	1	1	1	1	•	-	-	-	-
560-600	1	1	1	1	-	-	-	-	-
$\sum \left(\frac{\text{dpm}}{1000 \text{ SCF}}\right)$	338	343	431	440	366	299	155	138	112
Area' 108mi ²)	0.015	0.045	0.074	0.100	0.123	0.143	0.158	0.168	0.174
$(\sum)(Area)$	5, 1	15.4	31.9	44.0	45.0	42.8	24.5	23.2	19.5
		(251.4) (13.48×10	-4) = 0.3	4 mega	curie		



MOVEMENT OF NUCLEAR DEBRIS WITHIN THE STRATOSPHERE

A necessary preliminary to the preparation of a usable model of stratospheric transfer and mixing is the determination of the relative importance in
the stratosphere of such processes as turbulent exchange and organized transfer.
Important clues for such an evaluation are obtained by monitoring changes which
occur in the stratospheric distribution of nuclear debris and of natural tracer
nuclides. Observations of such changes during the course of the Star Dust program
have provided additional information to supplement that obtained during the High
Altitude Sampling Program. The new information has reinforced some of the
conclusions reached during HASP, but it has indicated that a few of these conclusions may require extensive modification.

We will consider first the evidence on stratospheric transfer processes derived from Star Dust measurements of fission product concentrations in samples collected during early 1962 and then the evidence derived from measurements of tracer nuclides in these samples.

Change with Time in the Distribution of Fission Products.

The mean distributions of total beta activity in the Star Dust sampling corridor during each of the last three months of 1961 and the first six months of 1962 have been shown either in the Fourth Quarterly Report² or in this report (see Figures 1 to 4), and the mean distributions of strontium-90 in the Star Dust sampling corridor during this same interval have been shown in Figures 5 to 8.

These distributions indicate that in January 1962 the highest stratospheric concentrations of debris from the 1961 Soviet tests were to be found at 50,000 to 60,000 feet altitude at high northern latitudes, and that this situation did not change during the first six months of 1962. Some vertical movement of this Soviet debris was occurring, however, as may be seen from the data in Figures 14 to 19.

At 65 °N there was a gradual decrease in strontium-90 activity at 50,000 feet during March to June 1962 (Figure 17) but the total beta activity did not decrease perceptibly (Figure 14), perhaps due to influx of fresh debris from low latitude United States tests. At 40,000 feet both strontium-90 and total beta activities showed a downward trend during these months. It is likely that debris from the United States tests had reached the 50,000 foot layer at 65°N but not the 40,000 foot layer because the sloping surfaces along which mixing between the tropical and polar stratospheres takes place do not reach down below 50,000 feet. Thus debris from tropical injections would have to mix vertically after entering the polar stratosphere before it could reach 40,000 feet at the high latitudes, at least during this season. Both total beta activities and strontium-90 concentrations rose markedly at 65,000 feet at 65°N during March to June 1962, but the debris entering this high layer was derived from the 1961 Soviet tests. Since it brought with it higher concentrations of antimony-124 than had previously been intercepted in the Star Dust corridor it must have come from a region which had not been sampled, presumably from the layers of air above 70,000 feet.

At 50 °N (Figures 15 and 18) the phenomena observed were similar to those noted above for 65 °N, except that both strontium-90 and total beta activities appeared to increase at 50,000 feet, at least by June 1962, doubtless as a result of influx of United States debris.

At 30°N (Figures 16 and 19) the total beta activity at 60,000 feet gradually decreased during January to April 1962, though the strontium-90 activity changed less noticeably. During May, however, the influx of debris from the 1962 United States tests produced an increase in concentrations at this altitude.

Activities at 50,000 feet fluctuated a good deal throughout early 1962, but the range in values was higher after the arrival of the United States debris in early May. Both the total beta and strontium-90 activities at 70,000 feet responded more slowly to the effects of United States tests, but a slow decreasing trend in concentrations during January to March did change to an increasing trend in early May, and by the middle of June activities were increasing rapidly.

Thus, during early 1962 no major changes were occurring in the distribution of nuclear debris in the northern polar stratosphere at altitudes of 40,000 feet and higher, though there was a gradual fallout of material from the lower layers and a gradual increase in activities at 70,000 feet due to vertical mixing of the 1961 Soviet debris. By May 1962 similar trends in the northern tropical stratosphere were obliterated by fresh debris resulting from the 1962 United States tests at Christmas Island, and some of this fresh debris also entered the polar stratosphere, especially at the 50,000 foot level.

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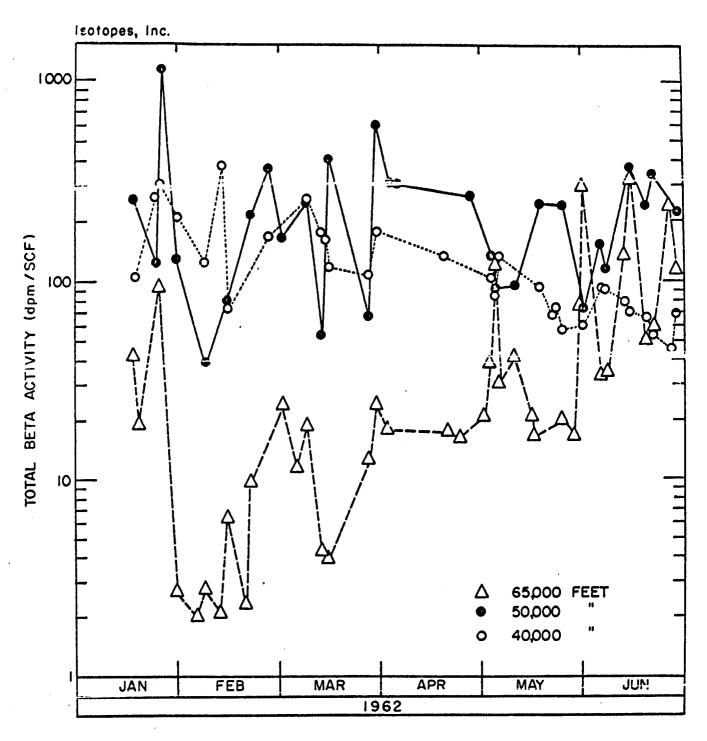


FIGURE 14 THE CHANGE WITH TIME OF THE TOTAL BETA ACTIVITY AT THREE ALTITUDES AT 65°N DURING 1962

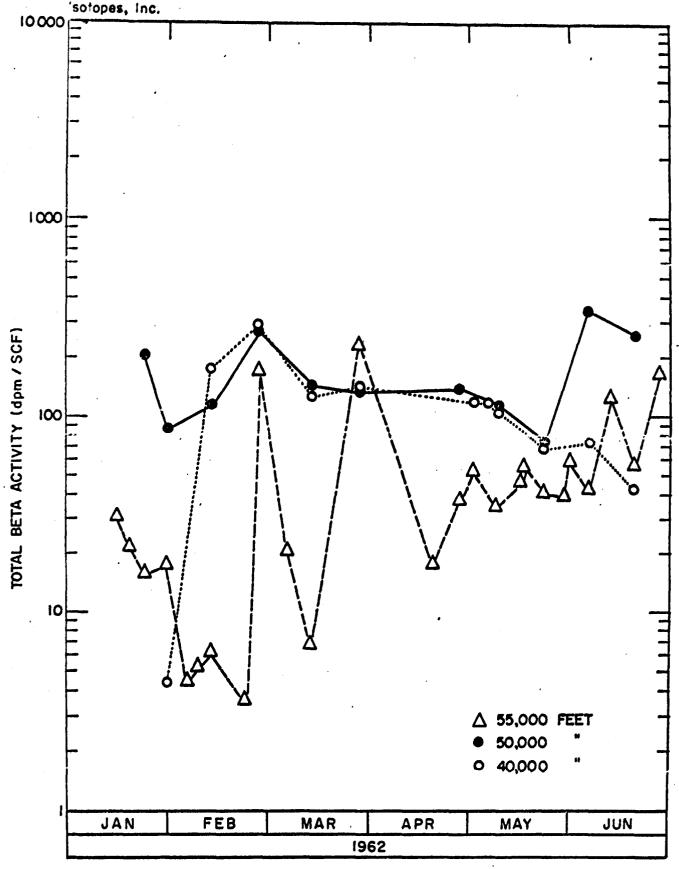


FIGURE 15 THE CHANGE WITH TIME OF THE TOTAL BETA ACTIVITY AT THREE ALTITUDES AT 50°N DURING 1962

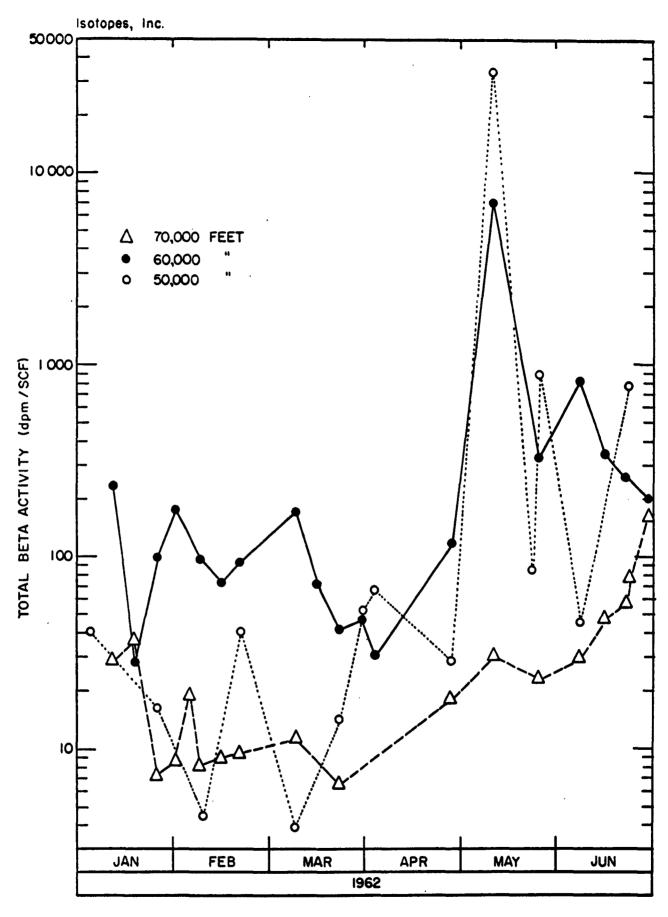


FIGURE 16 THE CHANGE WITH TIME OF THE TOTAL BETA ACTIVITY AT THREE ALTITUDES AT 30°N DURING 1962

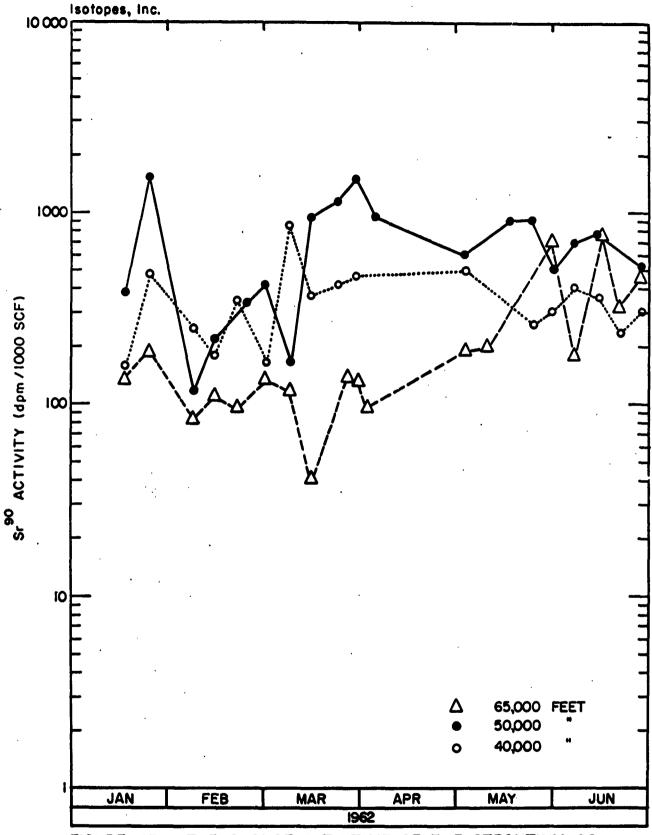


FIGURE 17 THE CHANGE WITH TIME OF THE STRONTIUM-90 ACTIVITY AT THREE ALTITUDES AT 65°N DURING EARLY 1962

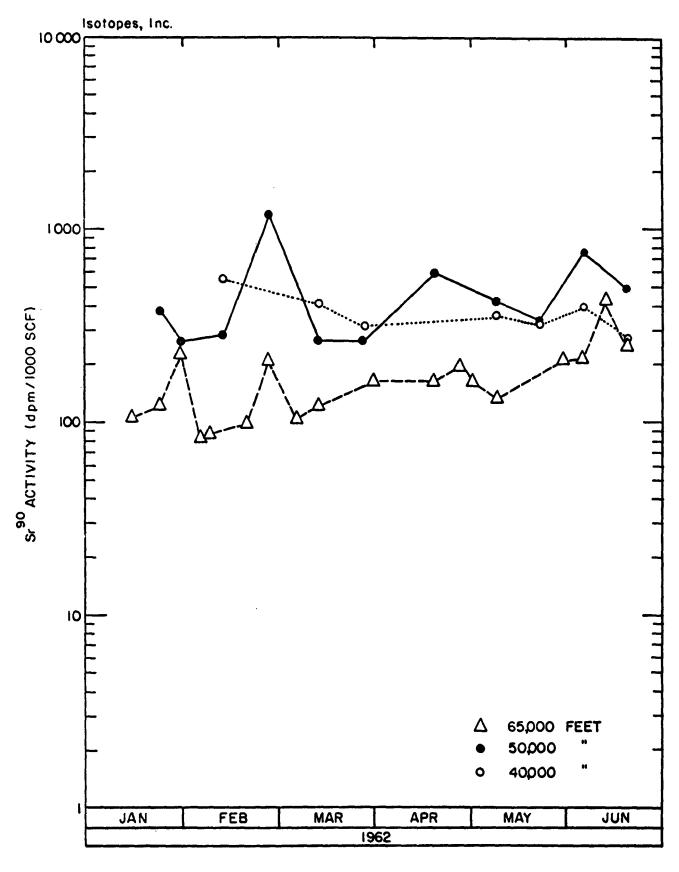


FIGURE 18 THE CHANGE WITH TIME OF THE STRONTIUM-90 ACTIVITY AT THREE ALTITUDES AT 50°N DURING EARLY 1962

Possibly one of the most significant aspects of the stratospheric distribution of strontium-90 during early 1962, especially as it relates to the transfer mechanisms active in the stratosphere, is the distribution shown in Figure 11 of strontium-90 from pre-1961 tests. This distribution was calculated from the strontium-90 concentrations and Sr89/Sr90 ratios found in the stratospheric debris. The change in the distribution of this old strontium-90 between June-September 1961 (Figure 10) and early 1962 is quite striking. It indicates that the layer of air containing the highest concentrations of old strontium-90, much of which was attributable to the 1958 rocket shots, Teak and Orange, had dropped from 70,000 feet or above to 60,000 feet and that a layer of air containing much lower concentrations had replaced it at 70,000 feet. This would appear to require an extensive seasonal organized transfer of stratospheric air which includes large scale subsidence in the northern polar stratosphere during the early winter months. There were indications in certain HASP data also that this might be true, but this new evidence, if it is borne out by further results, will be the strongest yet obtained during either HASP or Star Dust that organized motions do play an important role in stratospheric transfer. Further analysis of Ce^{144}/Sr^{90} ratios, Zr^{95}/Sr^{90} ratios and Sr^{89}/Sr^{90} ratios in the Star Dust samples will be required, however, before we can consider the interpretation given above to be completely justified.

Since it may be of interest to the reader to compare the concentrations of strontium-90 in the stratosphere during early 1962 with those found during

previous years, monthly averages of strontium-90 concentrations at a series of latitudes and altitudes, calculated from HASP and Star Dust data, have been plotted in Figures 20, 21, and 22.

In the northern polar stratosphere (Figures 20 and 21) the strontium-90 concentrations at 40,000 and 50,000 feet were higher during early 1962 than at any time that HASP measurements of these layers were being made, but at 65,000 feet concentrations were lower than they had been during 1959 and 1960. As was pointed out above, the high concentrations at 40,000 feet and especially at 50,000 feet probably reflect the relatively higher altitude of stabilization of clouds from the high yield weapons tested by the Soviets during October 1961 compared to the altitudes of stabilization of clouds from Soviet weapons tested during 1957 and 1958. It is unfortunate that HASP sampling was not performed at 40,000 feet in this region during 1958 or early 1959, but at that time it was not realized that debris from Soviet weapons tended to be concentrated at such low altitudes. It has also been pointed out above that the high activities at 65,000 feet during 1959 and 1960 resulted from United States tests, so that comparable activities in this layer might not be expected again until late 1962 or early 1963.

In the northern tropical stratosphere also (Figure 22) debris from the 1961 Soviet tests produced unusually high concentrations of strontium-90, at least at 50,000 and 60,000 feet. When debris from the 1962 United States tests also entered this region in May 1962 the concentrations reached values higher than any measured during HASP.

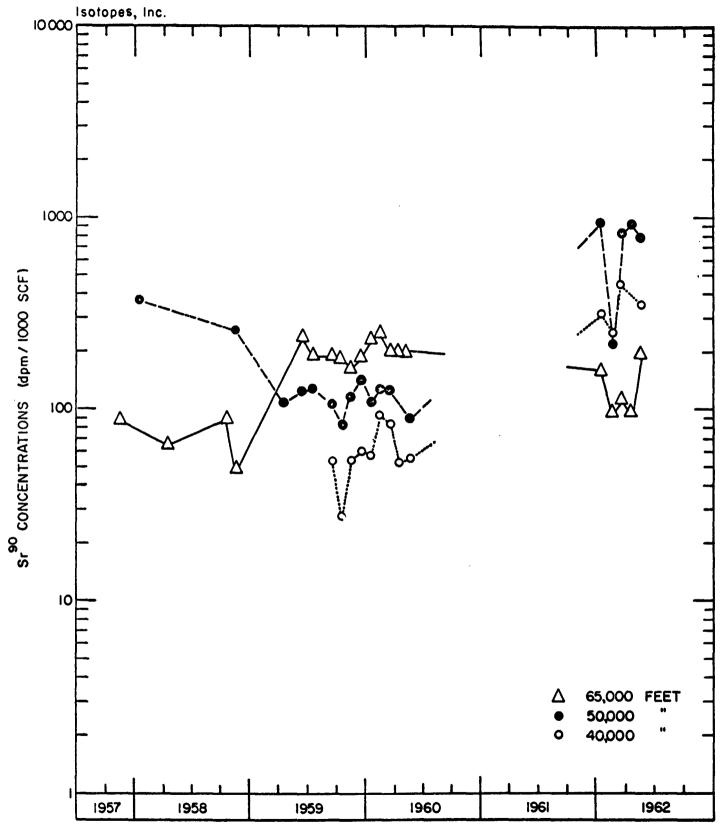


FIGURE 20 VARIATIONS WITH TIME IN THE MONTHLY AVERAGE STRONTIUM -90 CONCENTRATIONS AT THREE ALTITUDES AT 65°N

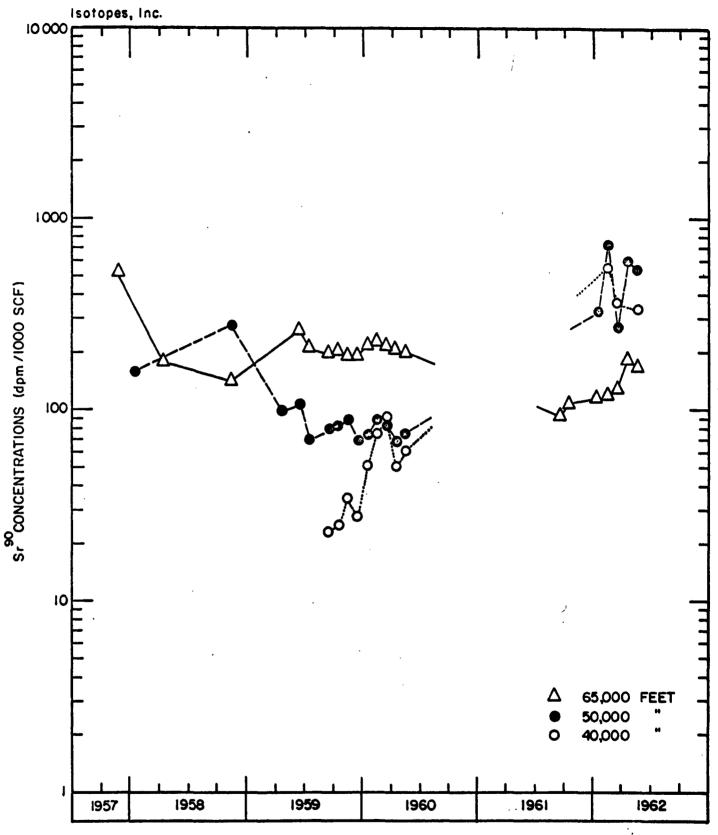


FIGURE 21 VARIATIONS WITH TIME IN THE MONTHLY AVERAGE STRONTIUM -90 CONCENTRATIONS AT THREE ALTITUDES AT 50°N

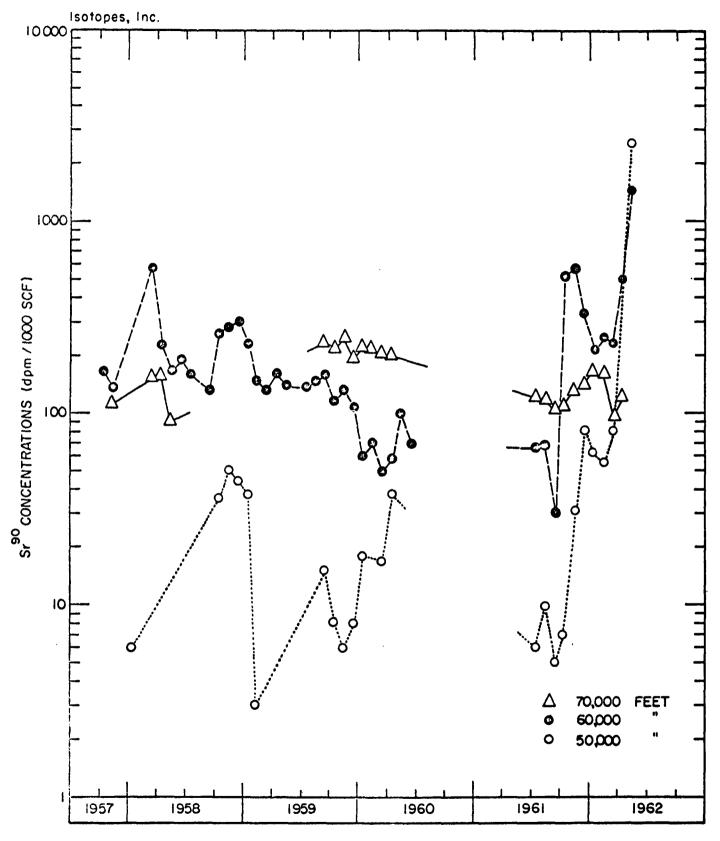
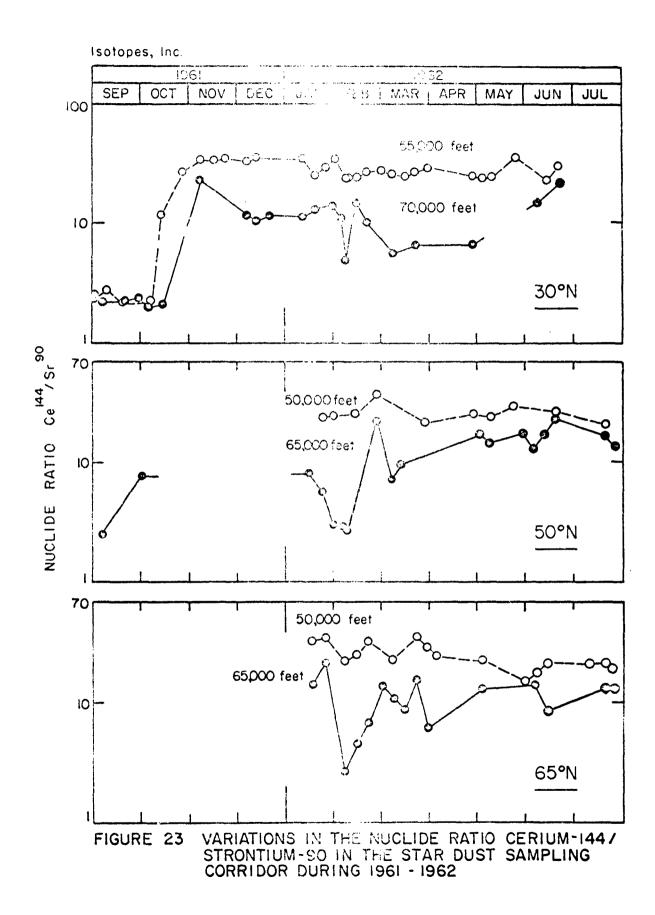
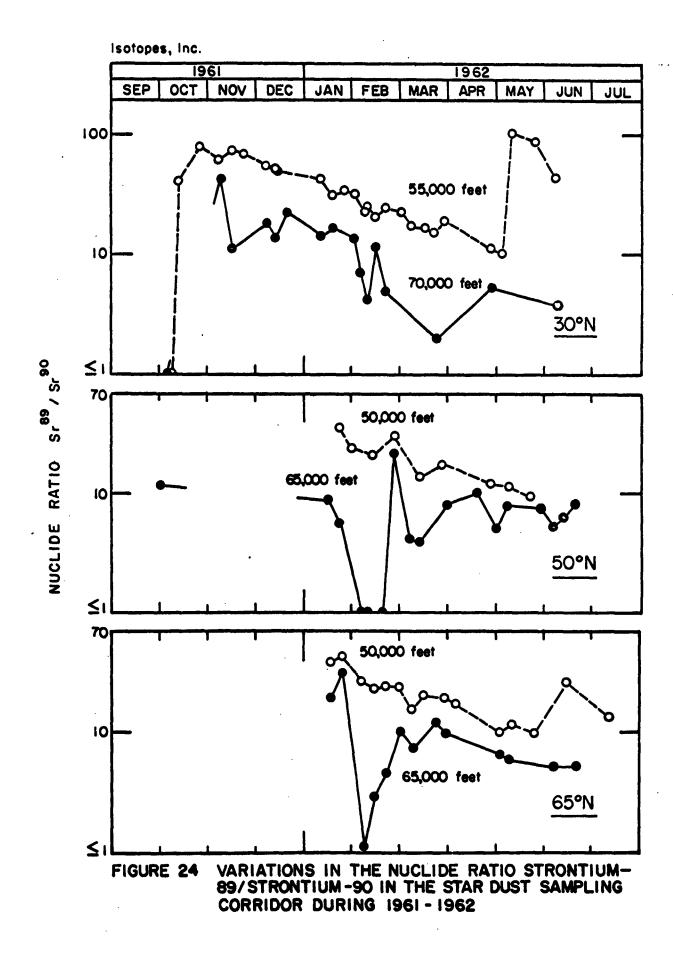
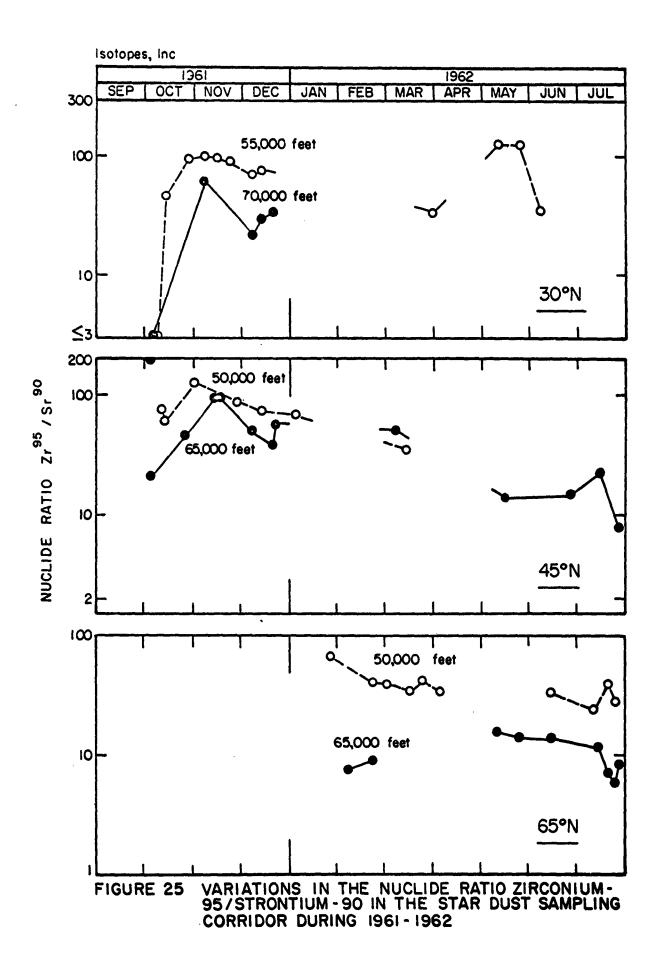


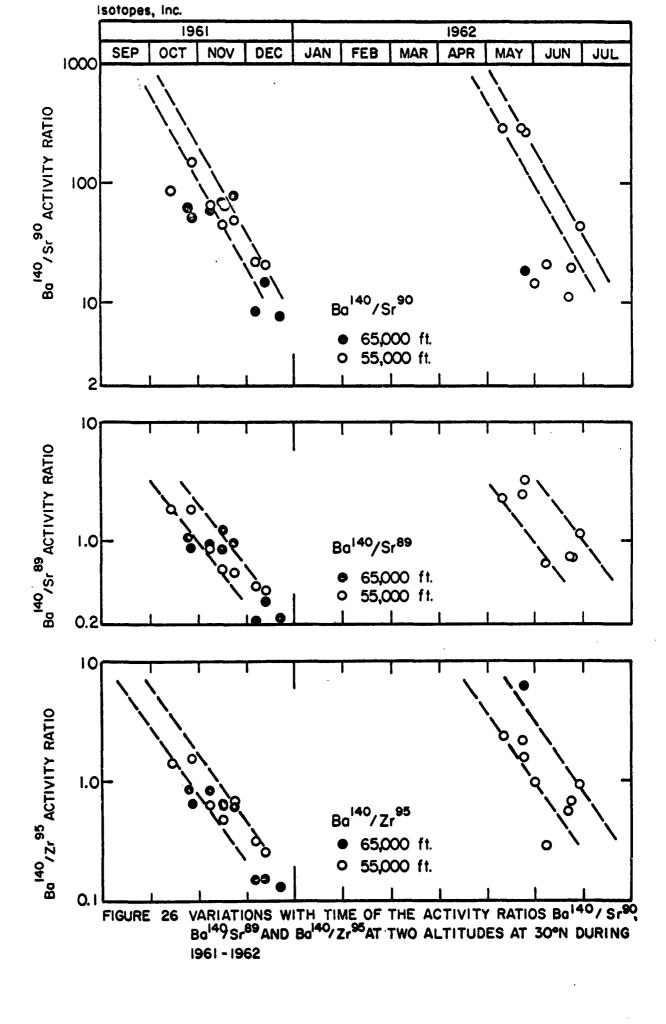
FIGURE 22 VARIATIONS WITH TIME IN THE MONTHLY AVERAGE STRONTIUM -90 CONCENTRATIONS AT THREE ALTITUDES AT 30°N

Figures 23 to 26 portray some of the changes observed in several fission product activity ratios during 1961 - 1962. During this entire interval the Ce¹⁴⁴/Sr⁹⁰ ratio (Figure 23), the Sr⁸⁹/Sr⁹⁰ ratio (Figure 24), and the Zr⁹⁵/Sr⁹⁰ ratio (Figure 25) were lower in debris at 65,000 or 70,000 feet than in debris at 50,000 or 55,000 feet in the Northern Hemisphere. This reflects the much lower concentrations of 1961 Soviet debris (and by mid-1962, of 1962 United States debris) at the higher altitudes. The failure of the figures to show fairly smooth decay curves is doubtless due mainly to incomplete mixing of old and new debris and of debris from all shots within the Soviet or within the United States test series. Insufficient data on Ba¹⁴⁰/Sr⁹⁰, Ba¹⁴⁰/Sr⁸⁹ or Ba¹⁴⁰/Zr⁹⁵ ratios (Figure 26) were available for latitudes other than 30 °N to make plotting them worthwhile.









Change with Time in the Distribution of Tracer Nuclides.

The "tracer" nuclides measured during Star Dust have included natural activities, such as beryllium-7, and artificial activities, such as rhodium-102, manganese-54 and antimony-124.

Three natural "tracer" nuclides have been measured in Star Dust samples, beryllium-7, lead-210 and polonium-210. Beryllium-7 is produced in the atmosphere by cosmic ray bombardment of nitrogen molecules. Lead-210 and polonium-210 are daughter products of radon, which is released to the atmosphere by the decay of radium in rocks, soil and waters at the earth's surface.

Bleichrodt⁵ has described the presence of artificial beryllium-7, produced by the 1961 Soviet test series, at 42,000 feet in the stratosphere during October-November 1961. He has estimated the relative concentrations of artificial beryllium-7 and strontium-89 to obtain a ratio which might be used to correct measurements of beryllium-7 in rainwater for their artificial component. The ratio which he has calculated, 0.034, is not inconsistent with data for Star Dust samples collected at 60,000 feet and below in the polar stratosphere during early 1962.

The variation with time of the concentrations of beryllium-7 and strontium-89 during 1961-1962 are shown in Figure 27 (for 65°N) and Figure 28 (for 30°N). In general the concentrations of these two nuclides vary together. This is attributable in part to the origin of some of the beryllium-7 in the weapon tests which produced the strontium-89. At low altitudes, however, parallel

variations in concentrations of both strontium-89 and beryllium-7 may result from mixing of stratospheric air, which contains high concentrations of both, with tropospheric air. At low latitudes (Figure 28), parallel variations during October 1961 - April 1962 in concentrations of these two nuclides could result from mixing of polar air, which contained high concentrations of both, with tropical air, which contained lower concentrations. Thus, even though strontium-89 measurements are available for the Star Dust samples, it is not easy to determine the ratio of artificial to natural beryllium-7 in the samples. It must be expected that this ratio will vary with latitude and altitude as well as with time. Thus the usefulness of this nuclide as a tracer for atmospheric transfer will be quite limited during periods of nuclear weapon testing.

The concentrations of lead-210 and polonium-210 found in HASP and Star Dust samples have been described previously 4 and have been compared with results reported by others. The main conclusions reached thus far have been that the stratospheric concentrations appear to be fairly constant at about 0.3 to 0.5 dpm Pb²¹⁰/1000 SCF and 0.3 to 0.5 dpm Po²¹⁰/1000 SCF and that the activity ratio Po²¹⁰/Pb²¹⁰ is about 1.0 in stratospheric air. This indicates a long atmospheric storage time for these nuclides, in excess of a year. There is some indication that activities of both nuclides decrease toward higher altitudes at higher latitudes. This is consistent with the origin of these nuclides in the troposphere. The interpretation of the data is limited by the precision of the measurements, however, and thus far it is not completely clear which of the

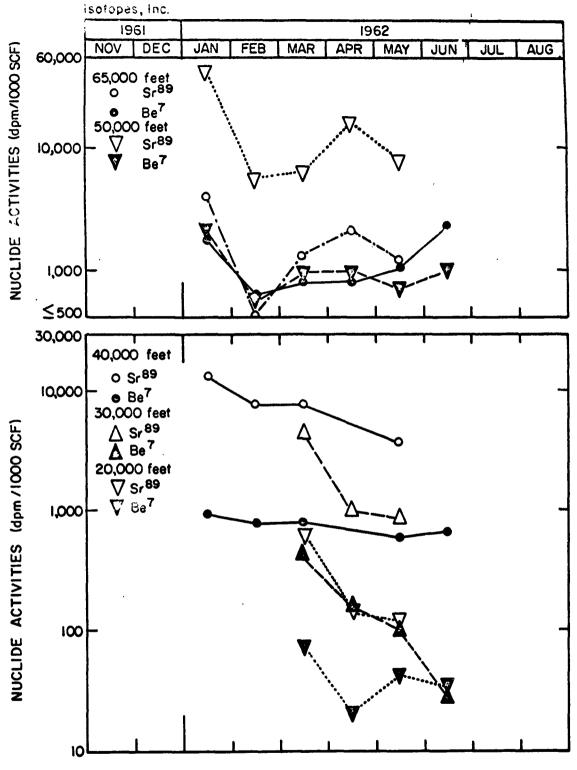


FIGURE 27 VARIATION WITH TIME DURING 1961-1962 OF BERYLLIUM-7 AND STRONTIUM-89 CONCENTRATIONS AT 65°N

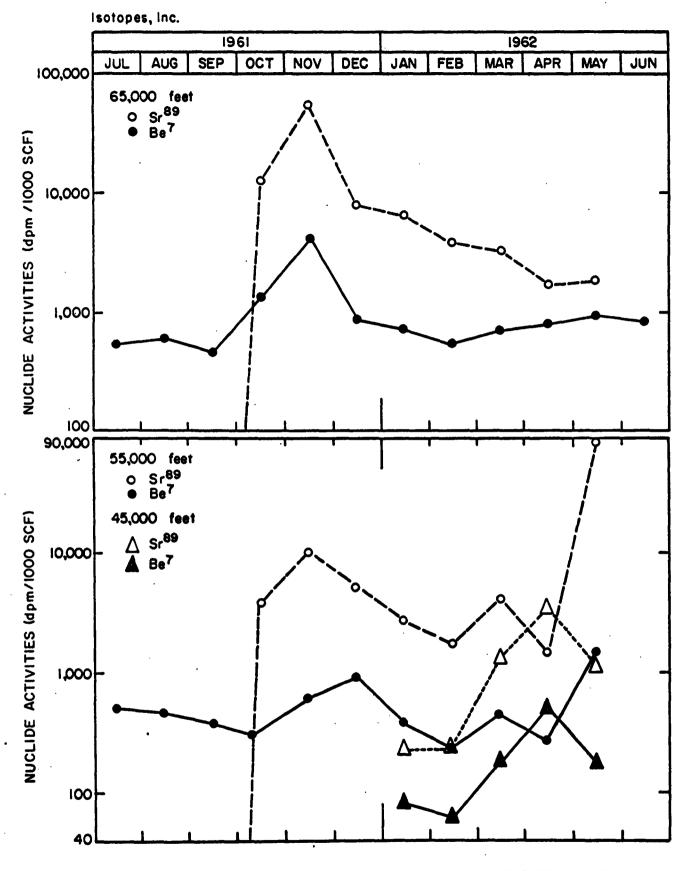


FIGURE 28 VARIATION WITH TIME DURING 1961-1962 OF BERYLLIUM-7 AND STRONTIUM-89 CONCENTRATIONS AT 30°N

observed variations in concentration are real and which are caused by analytical error.

Trends with time in the concentrations of lead-210 and polonium-210 at several latitudes are shown in Figures 29 through 31.

The artificial tracers which have been measured during Star Dust have included rhodium-102, produced by the 12 August 1958 United States Orange rocket shot, and manganese-54, cobalt-57, cobalt-58, iron-55 and antimony-124, attributable for the most part to the 1961 Soviet series.

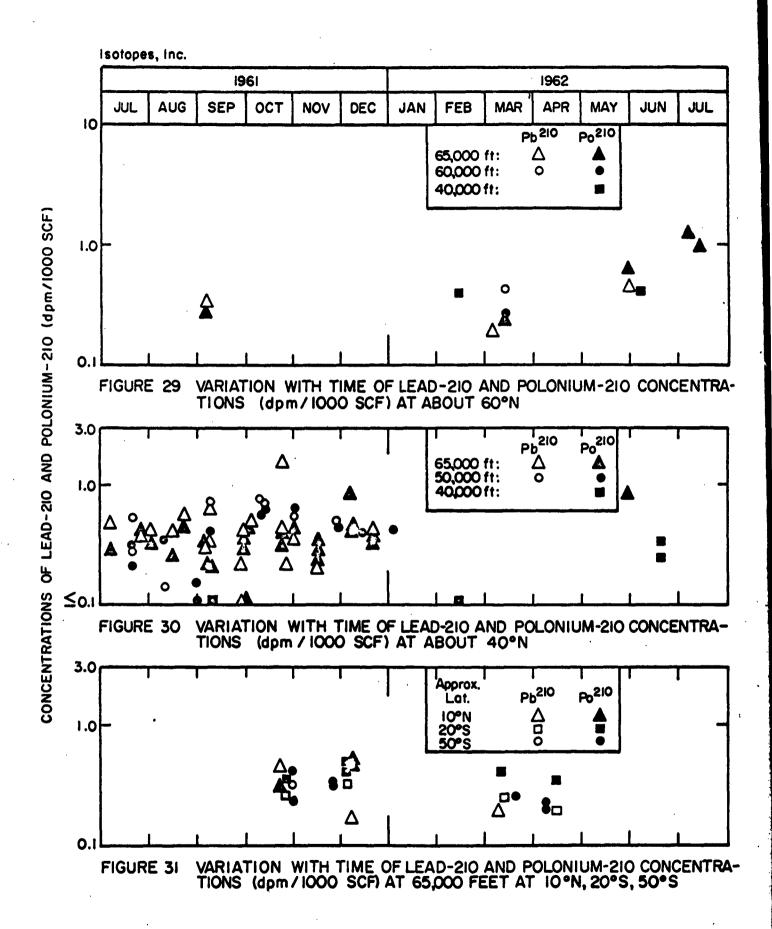
Rhodium-102 from the 1958 rocket shots was first detected in the lower stratosphere of the Northern Hemisphere in late 1959. Several laboratories have measured this nuclide in samples of stratospheric dust 4, 6, 7, 8, but the results of these analyses have not shown satisfactory agreement. Intercalibration between the laboratories is now underway and it is believed that much of the discrepancy will soon be removed. For the present, however, all HASP and Star Dust rhodium-102 data must be considered preliminary and it must be expected that future adjustments may result in an increase in the present numbers by 20% to 50%.

Beginning in January 1962 there was a reduction in the number of Star

Dust samples being analyzed for rhodium-102, so that data for 1962 are sparse.

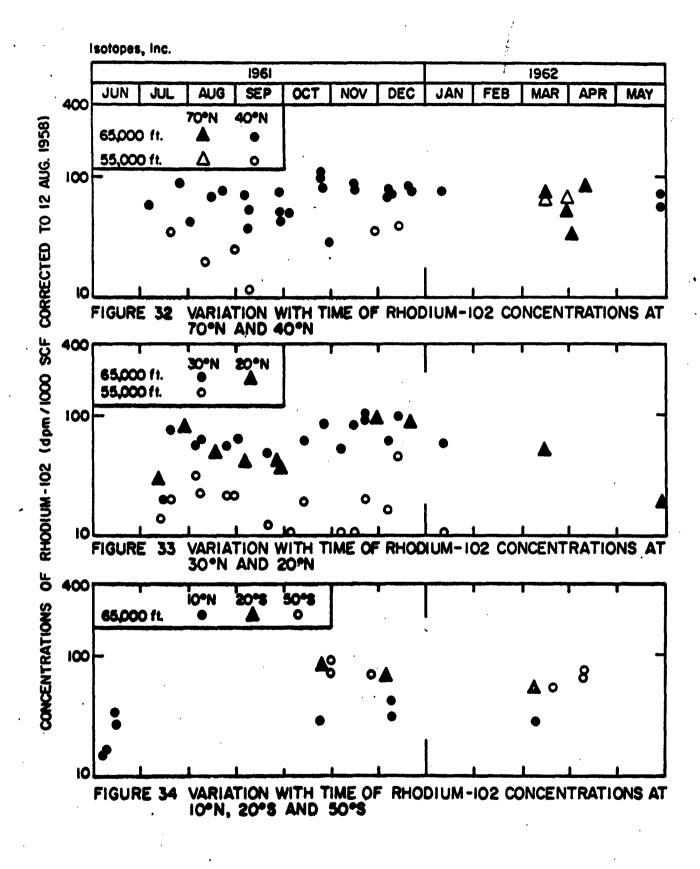
The results which we have, however, fail to indicate any major change in the concentration of rhodium-102 (corrected for decay) in the stratosphere during

1961 - 1962. Trends in this concentration at several latitudes and altitudes



are shown in Figures 32 to 34. On the basis of strontium-90 and Sr⁸⁹/Sr⁹⁰ data for January - April 1962 we would have predicted a significant drop in rhodium-102 concentrations at the higher altitudes in the northern polar stratosphere during the winter of 1961 - 1962. The data in Figure 32 do not substantiate this prediction, but the situation should be clarified as additional data become available.

Preliminary results of the analysis of Star Dust samples for manganese-54, cobalt-57, cobalt-58 and iron-55 are presented in Table 4. Much of the variability in the relative concentrations of the nuclides listed in the table is caused, no doubt, by differences in amounts produced by different weapons in the 1961 Soviet series. Other variations in the data are more difficult to explain. These measurements were more or less exploratory, for these tracer nuclides were not measured previously during HASP or Star Dust. A series of additional samples, chosen more systematically as regards latitude, altitude and time of collection, are now being analyzed for manganese-54, cobalt-57, iron-55 and antimony-124. Data for this series will aid in the determination of patterns in the relative concentrations of these tracers. Originally it was believed that debris from the high yield Soviet weapons, especially that from the high total yield, low fission yield device detonated on 30 October 1961, should display a high ratio of tracers to fission products, for the high neutron fluxes produced by these weapons should have led to unusually intensive neutron activation of the weapon components. According to this premise, much of the debris sampled in



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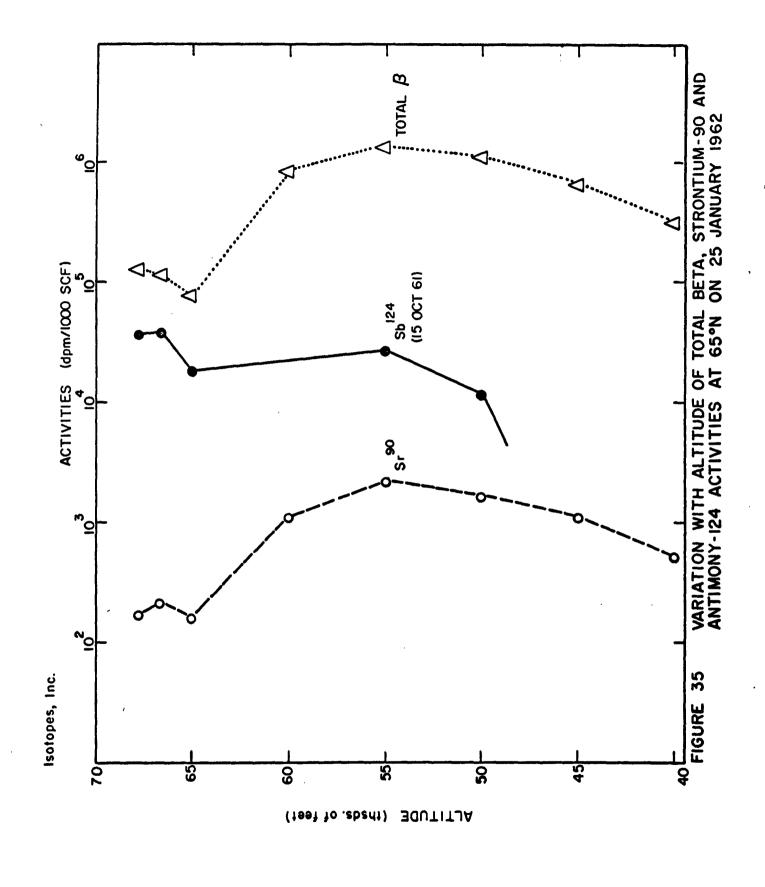
Table 4. Tracer nuclide concentrations in Star Dust samples.

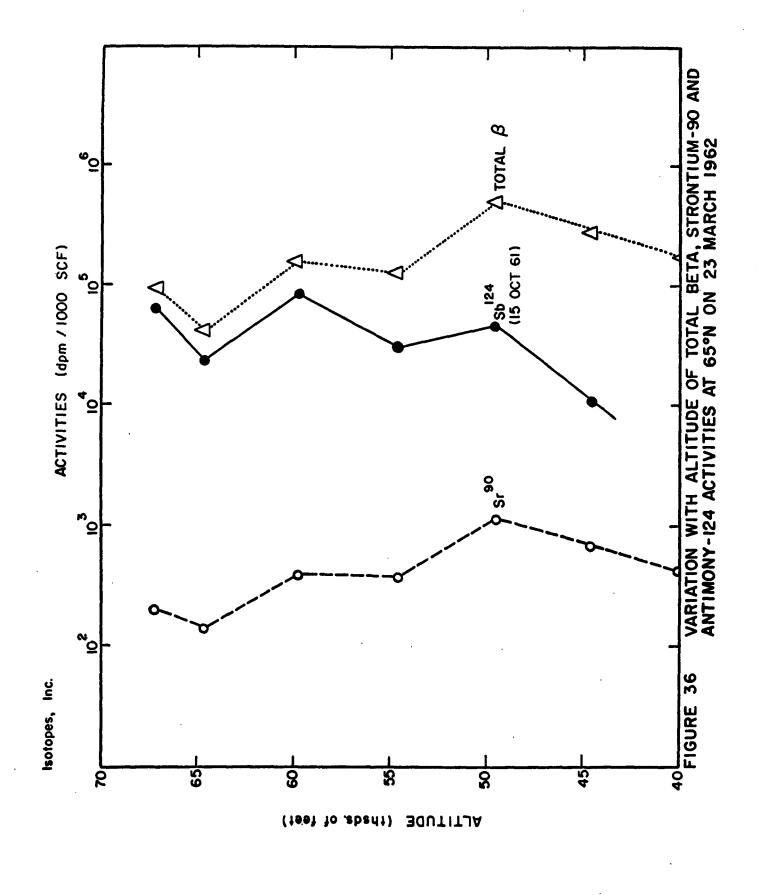
					Nuclide A	Nuclide Activities (corr. to 15 Oct. 1961)	orr, to 15	Oct. 1961)	
Collection Date	Sample No.	Lat.	Alt. (feet)	dpm B 1000 SCF	dpm Sr ⁹⁰ 1000 SCF	dpm Mn ⁵⁴	dpm Co ⁵⁷ 1000 SCF	dpm Co 1000 SCF	dpm Fe
5 Oct, 61		30 °N	60,000	1, 107,000	249	176	•	1	116
20 Oct, 61		15°-26°N	68,000	2,000	96	< 0.8	ı	ı	23
25 Oct. 61	SZ-2	28°-43°N	60,000	5, 227, 000	732	<u>9</u> 38	68	633	615
Oct.		N° 08	60,400	7,400,000	916	8,098	127	1,532	836
Oct.		30 °N	63,800	1, 586, 000	723	1,746	•	5	543
Oct.		28 - 38 °N	60,400	1, 296, 000	295	2,603		<1,400	589
Nov.		30 %	66, 400	3, 370,000	1,985	2,898	370	_< 128	1,539
Nov.		30 °N	65,600	4, 220, 000	2,522	4,611	3, 57 1	$\overline{2}$, 401	1,990
Dec.		38°-48°N	50,000	476,000	330	627	15	< 187	256
Dec.		28°-48°N	59,000	293,000	327	215	ı	1	237
Jan,		31°-54°N	55, 100	211,000	494	911	144	441	584
Jan.		53°-64°N	40,100	208,000	364	416	•	ŗ	308
Jan,		49°-64°N	9e, 800	12,000	125	121	27	•	-1
Jan,		N.0459	52, 500	1,097,000	1,523	4, 7-1	272	1,804	2,493
Jan.		Nº 02-059	45,300	652,000	1, 140	3.1	105	801	819
Feb.		50°-64°N	39,700	276,000	546	4,017	ı	•	419
Feb.		N. 02-, 59	54,900	80,000	276	471	42	546	435
Feb.		N. 49	51,300	284,000	531	3, 598	145	1,033	1,650
Mar		N° 64 - ° 04	65,600	419,000	608	58, 100	2,806	12,720	22,300
Mar		64°N	35,000	255,000	629	1, 101	64	516	405
Mar		64°N	65,400	36,000	200	6, 205	235	1,780	2,448
23 Mar. 62		N° 4 9	49,500	500,000	1, 162	12,621	664	3, 528	5,284
Mar		64°N	67,200	000'96	222	19, 126	781	7,694	7,648
19 Apr. 62	SZ-24	20°-63°N	64,500	33,000	246	5, 499	212	1,623	3, 266

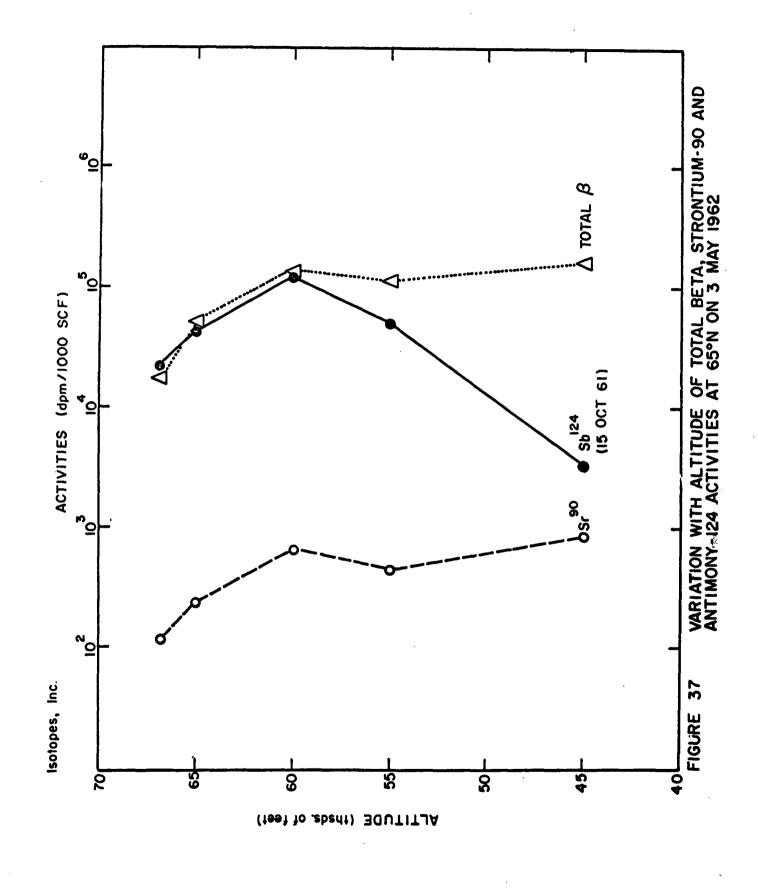
the northern polar stratosphere during March 1962 might be attributable to these tests.

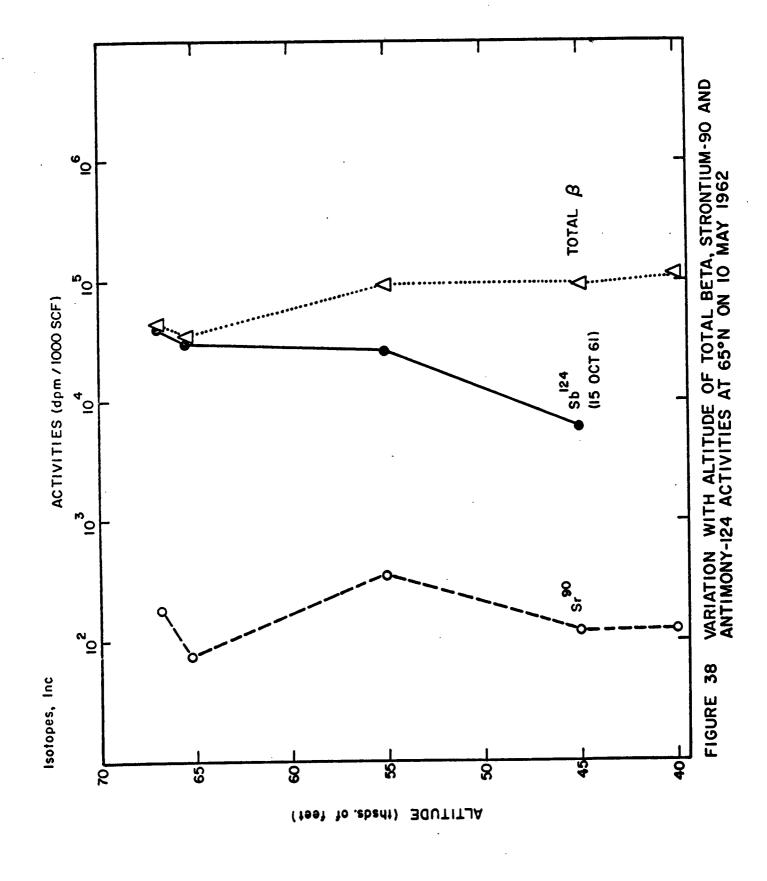
Antimony-124, the final tracer nuclide currently being studied in Star Dust, had apparently not been observed in significant concentrations in debris from weapon tests before the 1961 Soviet series. Peaks attributable to this nuclide were quite evident², however, in gamma spectra of many Star Dust samples collected during early 1962. In Figures 35 through 38 are plotted the vertical profiles at 65°N of total beta activity, strontium-90 activity and antimony-124 activity on four dates in early 1962. Although the antimony-124 activity varies with the total beta activity at the higher altitudes it falls off much more rapidly than does the total beta activity at the lower altitudes. This suggests that it had a higher mean altitude of injection than did the bulk of the debris from the 1961 Soviet tests, and probably originated in one or more of the higher yield weapons. It is also noteworthy that the highest concentrations of this nuclide found thus far during Star Dust were in samples collected mainly during May and June 1962. Very high concentrations were also found in some samples collected on 6 March 1962. It is quite possible that debris containing high antimony-124 activities was entering the lower northern polar stratosphere from the layers of air above 70,000 feet during the spring of 1962. It is difficult otherwise to explain why antimony-124 concentrations in the Star Dust sampling corridor were generally quite low during January and February 1962, somewhat higher during March

and April and much higher during May and June. Again it is expected that continuing measurement of this nuclide in Star Dust samples will contribute greatly to our understanding of processes of stratospheric transfer.









MOVEMENT OF NUCLEAR DEBRIS INTO THE TROPOSPHERE

In the preparation of the Star Dust model of atmospheric mixing it is necessary to make assumptions regarding the relative importance of the various mechanisms of exchange of air between the stratosphere and troposphere.

During early 1962 the Star Dust sampling network was expanded and provision was made for the collection of some samples in the vicinity of the tropopause.

Unfortunately, the intensity of this sampling is insufficient to yield the type and quantity of data needed to perform a definitive analysis of atmospheric processes in this region. Nevertheless, the data obtained do provide clues to the nature of these processes.

A series of filter samples were collected in the vicinity of the polar tropopause at 65°N during March to June 1962. The total beta and strontium-90 activities of these samples are plotted in Figures 39 and 40 to show the variation of activity with time. The beta activity decreased at all four altitudes sampled during this interval, partly as the result of radioactive decay of the debris and partly as a result of fallout of debris to the lower atmosphere and the ground.

Strontium-90 concentrations at 20,000 and 30,000 feet alternately fell and rose, more or less in phase, but there was an overall downward trend. At 30,000 feet there were also wide fluctuations in strontium-90 concentration, but apparently no downward trend was actually established until May 1962. Concentrations at 35,000 feet showed much less fluctuation and it was not until June 1962 that a downward trend became evident.

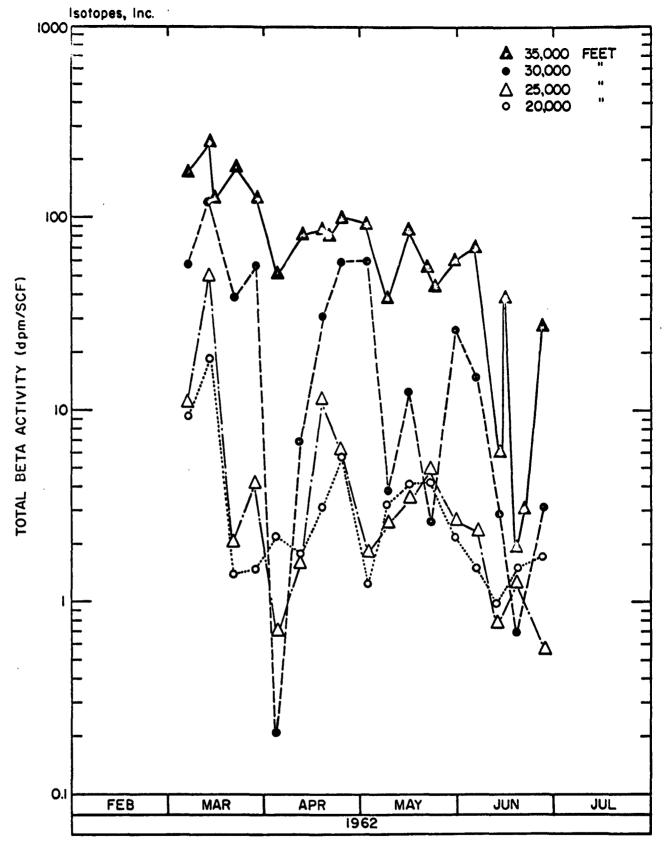


FIGURE 39 THE CHANGE WITH TIME OF THE TOTAL BETA ACTIVITY AT FOUR ALTITUDES NEAR THE TROPOPAUSE AT 65°N DURING 1962

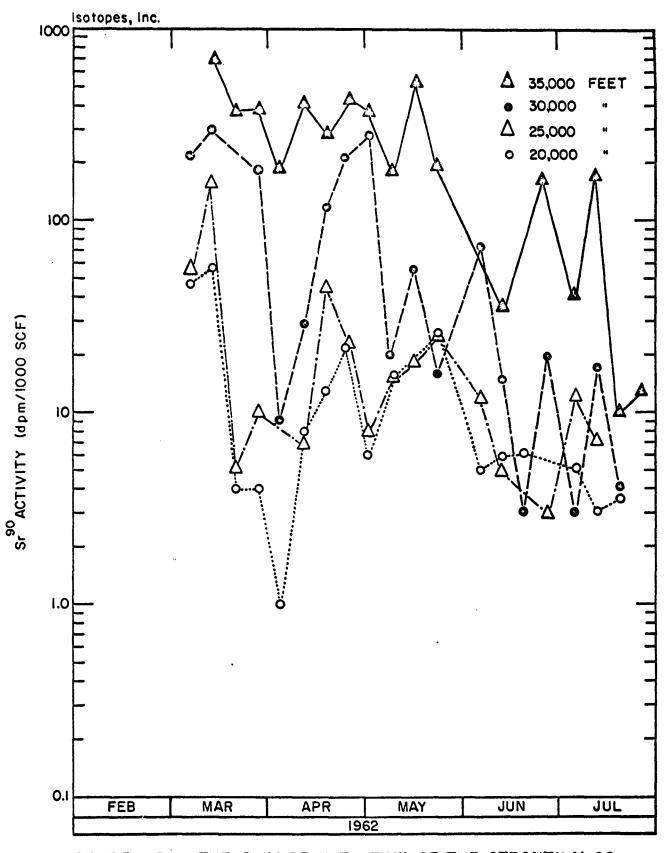


FIGURE 40 THE CHANGE WITH TIME OF THE STRONTIUM-90 ACTIVITY AT FOUR ALTITUDES NEAR THE TROPO-PAUSE AT 65°N DURING 1962

A measure of caution must be used in evaluating these data. They do appear, however, to be quite consistent with the hypothesis which assigns to the spring increase in the height of the tropopause a major role in the transfer of the debris from the stratosphere to the troposphere. The decreasing trend in strontium-90 concentrations was evident at 20,000 and 25,000 feet by April, at 30,000 feet by May and at 35,000 feet by June 1962.

Data for samples collected near or below the tropical tropopause at 30 °N during the first half of 1962 are illustrated in Figures 41 and 42. There is an increasing trend in both total beta activity (Figure 41) and strontium-90 activity (Figure 42), though wide fluctuations in concentration occurred from mission to mission. The increases during May - June 1962 are attributable to influx of young debris from the United States weapon tests at Christmas Island which began in April 1962. Even before this fresh debris entered the Star Dust sampling corridor at 50,000 feet and above during early May, however, concentrations of debris at 40,000 and 45,000 feet had shown a pronounced increase. This can be attributed only to the southward movement through the tropopause gap region of debris from the 1961 Soviet test series.

Vertical profiles of beta activity at 30 °N are shown in Figures 43 to 45 for several sampling dates during early 1962. On most of these sampling dates the equatorward extension of the polar tropopause was present at about 43,000 feet and the tropical tropopause was present at about 55,000 feet, as

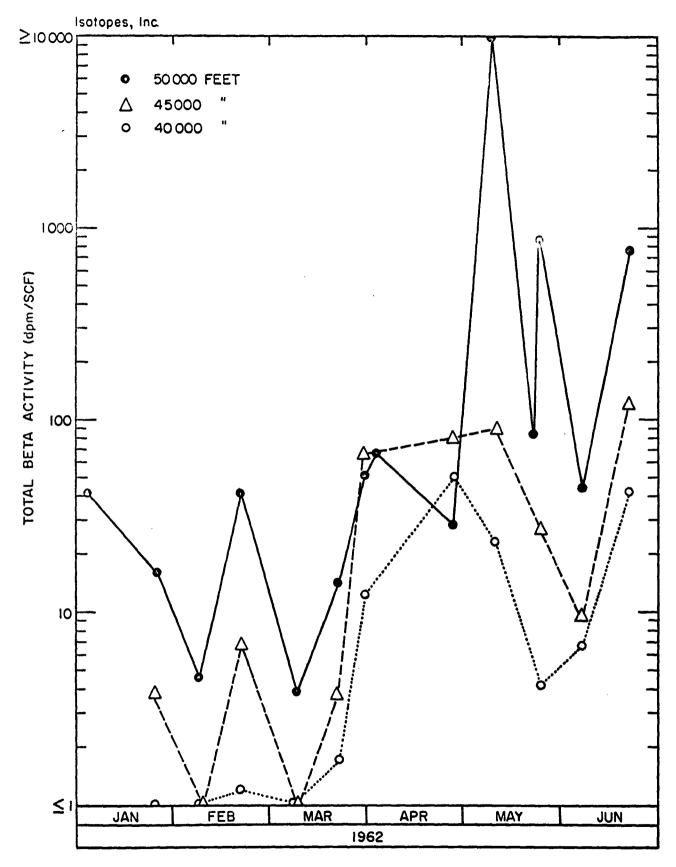


FIGURE 41 THE CHANGE WITH TIME OF TOTAL BETA ACTIVITY AT THREE ALTITUDES NEAR OR BELOW THE TROPICAL TROPOPAUSE AT 30°N DURING 1962

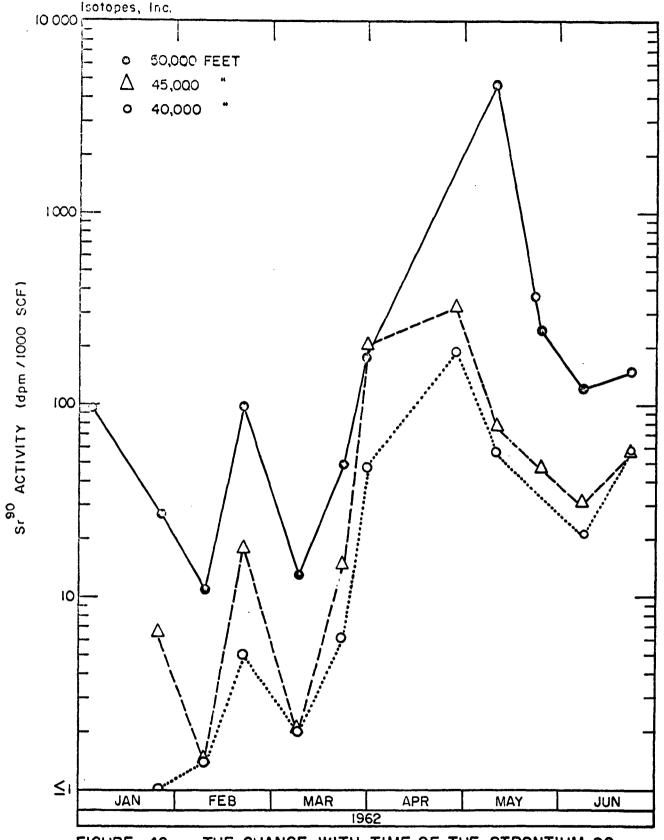
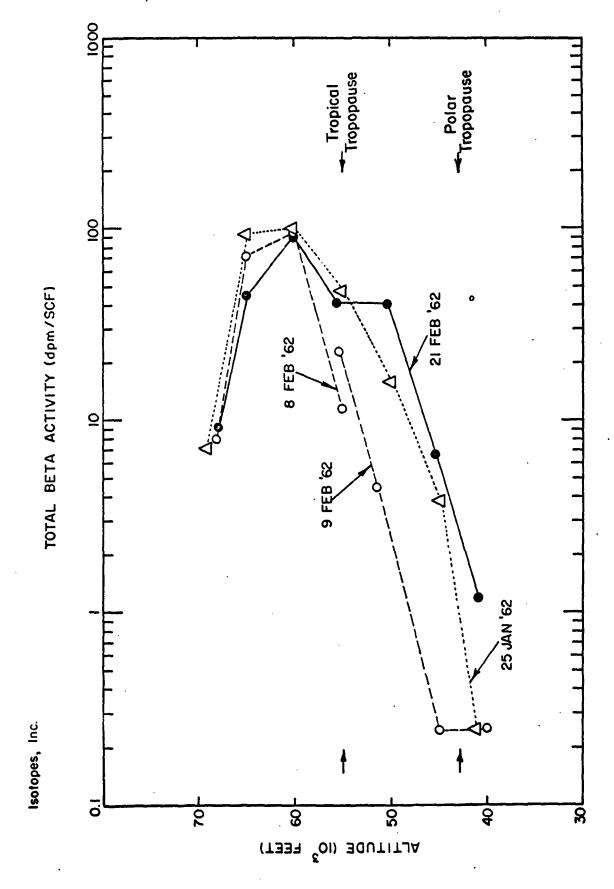


FIGURE 42 THE CHANGE WITH TIME OF THE STRONTIUM-90 ACTIVITY AT THREE ALTITUDES NEAR OR BELOW THE TROPICAL TROPOPAUSE AT 30°N DURING 1962

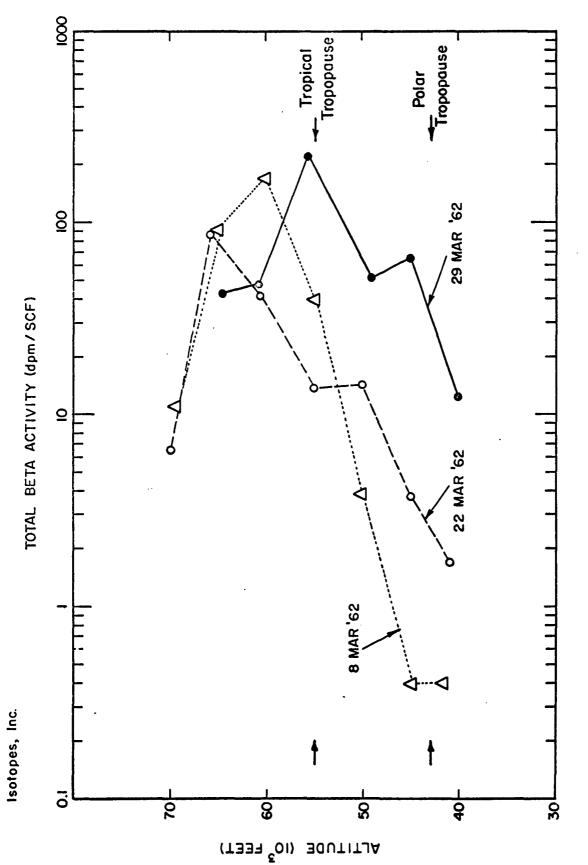
indicated by the arrows on the Figures. Up until 10 May 62, when debris from United States tests was encountered at 50,000 to 60,000 feet, the highest activities were always found in the layer immediately above the tropical tropopause. During late March, during April and during early May, however, relatively high activities were also encountered in the layer between the lower and upper tropopause. The hypothesis that debris from the 1961 Soviet tests was moving southward through the tropopause gap region and into the troposphere during the spring of 1962 is quite consistent with these observations.

It appears most likely that the increase in fallout rate which occurs each year during the spring months in the Northern Hemisphere is attributable in part to the rising of the polar tropopause during those months, converting stratospheric air into tropospheric, in part to increased lateral mixing through the tropopause gap region, and doubtless in part to other phenomena. The effect on the rate of fallout of the rising tropopause is probably greatest during years such as 1959 and 1962 which follow extensive injections of debris into the lowest layers of the stratosphere by Soviet tests. The effect of lateral mixing through the tropopause gap region (or through the sloping polar tropopause) is probably of the greatest significance in years such as 1960 and 1961 when the lowest layers of the polar stratosphere (at about 30,000 to 40,000 feet) contain concentrations of debris which are low compared to those in the overlying layers (at about 45,000 to 55,000 feet).

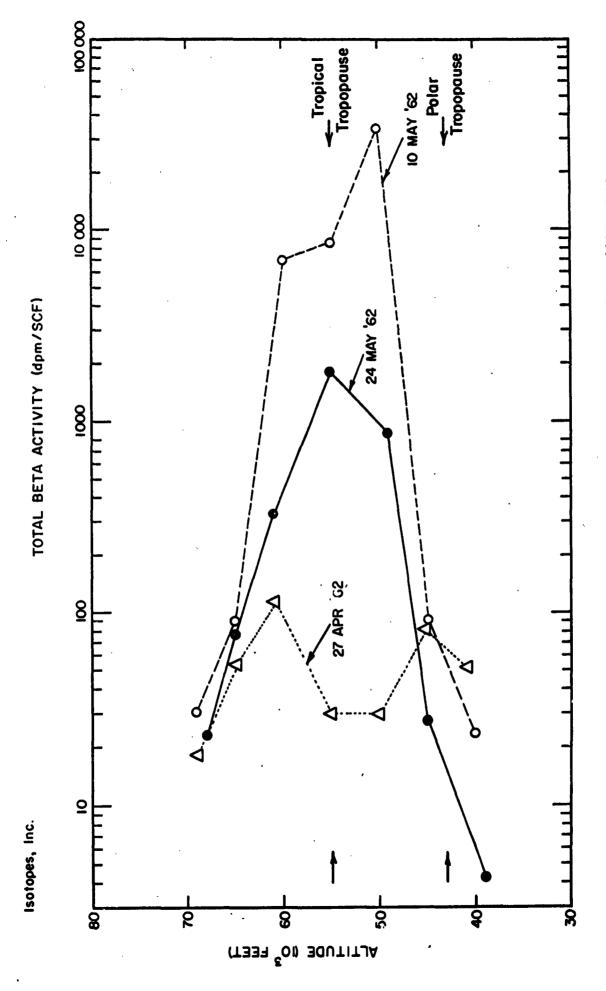
These conclusions concerning the movement into the troposphere of debris from nuclear weapons tests neglect any reference to processes such as those described by Danielsen and Staley 10, for the very precise, meteorologically controlled flight paths needed to test these hypotheses have not been possible as part of the Star Dust sampling program. Thus we can offer no evidence for or against these hypotheses.



VERTICAL PROFILES OF BETA ACTIVITY AT 30°N, JANUARY-FEBRUARY 1962 FIGURE 43



VERTICAL PROFILES OF BETA ACTIVITY AT 30°N, MARCH 1962 FIGURE 44



VERTICAL PROFILES OF BETA ACTIVITY AT 30°N, APRIL - MAY 1962 FIGURE 45

THE DESIGN OF THE STAR DUST MODEL

The basic philosophy adopted in this phase of Project Star Dust consists of treating the task as a set of mathematical (numerical) experiments in the earth - atmosphere system. The <u>guide lines</u> in these experiments are sets of observations such as distributions of trace materials in the stratosphere, distributions of the surface depositions of nuclear debris, rates of deposition of nuclear debris, and concentrations of trace materials in ground level air. Thus the efforts in this work will be directed toward finding a model which will be in agreement with the observed quantities. The <u>goal</u> of this work is to find a model with which predictions may be made of fallout patterns, rates, and attendant doses and dose rates resulting from various types injections of nuclear debris. The type of injection is characterized by latitude, altitude, and time of year. The goal will be reached when the model gives results in agreement with observation. The reliability of the predictions will, to a large degree, be related to the degree with which the model is in agreement with the various observable quantities listed above.

The Diffusion Equation Related to the Earth - Atmosphere System

The justification for using eddy diffusion as a basis for the model of atmospheric transport of trace materials has been presented in the Fourth Quarterly Report on Project Star Dust. 2 While it is presently believed that eddy diffusion can account for the gross features of the observed distributions

of nuclear debris, it is felt that the equations which describe the model should contain advective terms. This will permit assessment of the types and magnitudes of effects of various schemes which include meridional circulations. This is in accord with the above described philosophy. In addition to the advection and eddy diffusion terms, a term which represents the vertical velocity of the diffusing particles in quiet air as included. Thus the removal of material from the atmosphere by rain can be formulated by using a fictitous vertical velocity in the lower levels of the troposphere. The instantaneous flux of diffusing material through a closed surface in space is then given by

(1)
$$\mathbf{F} = (\mathbf{V} + \mathbf{\Omega}) \mathbf{Q}$$

where the underline is used to denote a vector quantity and

Q = concentration of diffusing material;

$$g/cm^3$$

V = instantaneous velocity of the air

 Ω = instantaneous velocity (vertical) of particles relative to air.

The equation of continuity for the diffusing material is

(2)
$$\frac{\partial Q}{\partial t} = - \text{Div } \underline{F}$$

where t is time.

The quantities V and Q are now expressed in terms of averages:

(3)
$$\underline{V} = \overline{\underline{V}} + \underline{V}'$$
, $Q = \overline{Q} + Q'$

Here all quantities are now considered to be zonally averaged (in the earth - atmosphere system) and

V = component of velocity of air due to its (time) mean motion.

 \underline{V}^{t} = component of velocity of air due to its eddy motions.

 \overline{Q} = time average concentration of diffusing material.

Q'= concentration of diffusing material due to the eddy motions of air.

This term is actually defined by $Q' = Q - \overline{Q}$.

Substituting (3) in (2) gives

(4)
$$\frac{\partial \overline{Q}}{\partial t} + \text{Div} \left[(\overline{\underline{V}} + \underline{\Omega}) \overline{Q} \right] = \text{Div} (\overline{\underline{V'}} Q).$$

Equation (4) contains no Ω term since $\underline{\Omega}$ is defined as having no eddy component, and thus $\underline{\overline{\Omega}} = \underline{\Omega}$.

The coefficient of eddy diffusion, K, is defined empirically by

(5)
$$\underline{F}$$
 eddy = $-\rho K \nabla q$

where \underline{F} eddy is the eddy flux of material K is a tensor of rank two, ρ = density of air; g/cm^3 , and

 $q = \frac{\overline{Q}}{P}$ = mixing ratio of diffusing material; g/g.

In this treatment of the atmosphere it is assumed that mixing in the vertical direction is not correlated with mixing in the meridional direction. This is equivalent to assuming that the coordinates chosen are the principle axes of the tensor, K. Thus, in spherical coordinates with zonal symmetry we have

(6)
$$K = \begin{pmatrix} K_r & 0 \\ 0 & K_M \end{pmatrix}$$

where r is the radial distance from the center of the earth and μ = $\sin \phi$ denotes the meridional direction with ϕ as latitude. (That is, a unit vector in the μ - direction, $\underline{\mathbf{u}}_{\mu}$, is perpendicular to a unit vector in the r - direction, $\underline{\mathbf{u}}_{\Gamma}$.)

Substituting (5) into (4) and ρq for \overline{Q} gives

(7)
$$\frac{\partial (\rho_q)}{t} + \text{Div} \quad (\overline{V} + \underline{\Omega}) \rho_q = \text{Div} (\rho K \nabla_q).$$

The equation of continuity for the air is

(8) Div
$$(\rho \underline{V}) = \frac{\partial \rho}{\partial t} = 0$$

This, in conjunction with (7) gives

(9)
$$\rho \frac{\partial q}{\partial t} + \rho \overline{V} \cdot \nabla q + \text{Div } (\Omega \rho q) = \text{Div } (\rho K \nabla q).$$

We now evaluate the quantities ∇q and the right hand side of (9) in spherical coordinates (remembering that all quantities are zonally averaged):

(10a)
$$\nabla_{q} = \underline{u}r \frac{\partial q}{\partial r} + \underline{u}\mu \frac{\partial q}{\partial \phi}$$

(10b) Div
$$(\rho K \nabla_q) = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \rho K_r \frac{\partial q}{\partial r} \right) + \frac{1}{r \cos \phi} \frac{\partial}{\partial \phi} \left(\frac{\rho K_{\mu}}{r} \frac{\partial q}{\partial \phi} \cos \phi \right)$$

The first term on the right in (10b) can be expressed as:

(11a)
$$\frac{\partial}{\partial r} \left[\rho K_r \frac{\partial q}{\partial r} \right] + \frac{2}{r} \rho K_r \frac{\partial q}{\partial r}$$

Since r is of the order of the radius of the earth (6.37x10⁸ cm), it can be seen that the second term in (11a) is negligible compared to the first term. The second term on the right of (10b) becomes exactly

(11b)
$$\frac{\rho}{r^2} \frac{\partial}{\partial \mu} \left[(1 - \mu^2) K_{\mu} \frac{\partial q}{\partial \mu} \right]$$

when we have

$$\mu = \sin \phi$$

 $\frac{\partial P}{\partial P}$ 0. (The density of air is assumed to be a function of r only.)

With the further approximation that $r \approx r_0$ in (11b) equation (9) now

may be written as

(12)
$$\frac{\partial q}{\partial t} = w \frac{\partial q}{\partial r} + \frac{1}{\rho} \frac{\partial}{\partial r} (\rho \Omega q) + v \cos \phi \frac{\partial q}{\partial (r_0 \mu)} = \frac{1}{\rho} \frac{\partial}{\partial r} (\rho K_r \frac{\partial q}{\partial r}) + \frac{\partial}{\partial (r_0 \mu)} \left[(1 - \mu^2) K_{\mu} \frac{\partial q}{\partial (r_0 \mu)} \right]$$

where (for convenience we list all quantities)

w = average vertical velocity of air around a latitude belt

 ρ = density of air, assumed to be a function of r only (gms/cm³).

The variation of density with height is assumed to follow the ARDC Model Atmosphere, 1959.

 K_r = diffusion coefficient in the r direction (cm²/sec)

 K_{μ} = diffusion coefficient in the north - south direction (cm²/sec)

 Ω = vertical velocity of diffusing particles in quiet air

v = zonally average poleward velocity (positive towards North Pole)

φ = geographical latitude

r * coordinate in the vertical direction

 $\mu = \sin \phi$

 r_0 = mean radius of earth - atmosphere system = 6.371x10⁸ cm

rou = coordinate - poleward direction

Numerical Solutions of the Diffusion Equation

An implicit, alternating direction method of numerically integrating equation (12) was chosen because of the inherent stability and numerical convenience associated with this method 11. The finite difference versions of equation (12)

associated with the follows:

13(a)
$$\frac{q^{n+1}-q^n}{8!} + \frac{1}{2} \left(\omega \frac{\delta q}{\delta r}\right)^n + \frac{1}{2} \left(\omega \frac{\delta q}{\delta r}\right)^{n+1} + \frac{1}{2\rho} \left(\frac{\delta}{\delta r}(\rho \Omega q)\right)^n + \frac{1}{2\rho} \left[\frac{\delta}{\delta r}(\rho \Omega q)\right]^{n+1} + \left(v \cos \phi \frac{\delta q}{r_0 \delta \mu}\right)^n = \frac{1}{2\rho} \left[\frac{\delta}{\delta r} \left(\delta K_r \frac{\delta q}{\delta r}\right)\right]^n + \frac{1}{2\rho} \left[\frac{\delta}{\delta r} \rho K_r \frac{\delta q}{\delta r}\right]^{n+1} + \left(\frac{\delta}{r_0^2} \frac{\delta q}{\delta \mu} \left[\left(1-\mu^2\right) K_\mu \frac{\delta q}{\delta \mu}\right]\right)^n$$

13 (b)
$$\frac{q^{n+1}-q^{n}}{8!} + \left(\omega \frac{8q}{8r}\right)^{n} + \frac{1}{\rho} \left[\frac{8}{8r} (\rho \Omega q)\right]^{n} + \frac{1}{2} \left[v \cos \phi \frac{8q}{r_{0}8\mu}\right]^{n} + \frac{1}{2} \left[v \cos \phi \frac{8q}{r_{0}8\mu}\right]^{n} + \frac{1}{2} \left[v \cos \phi \frac{8q}{r_{0}8\mu}\right]^{n+1} = \frac{1}{\rho} \left[\frac{8}{8r} (\rho \kappa_{r} \frac{8q}{8r})\right]^{n} + \frac{1}{2} \left[\frac{8}{r_{0}^{2}8\mu} \left((1-\mu^{2}) \kappa_{\mu} \frac{8q}{8\mu}\right)\right]^{n+1} + \frac{1}{2} \left[\frac{8}{r_{0}^{2}8\mu} \left((1-\mu^{2}) \kappa_{\mu} \frac{8q}{8\mu}\right)\right]^{n+1}$$

Where the superscripts indicate values of the quantities at time n and time n + 1. It is apparent that equation (13a) is implicit in the "r" direction and explicit

in the µ direction while equation (13b) is implicit in the µ direction and explicit in the "r" direction. Hence the term alternating direction. Equations (13a) and (13b) are to be solved in sequence. For the rest, centered finite differences are used to approximate the derivatives. For example, the most complicated first order difference:

$$\frac{1}{\rho} \frac{\delta}{\rho r} (\rho \Omega q) = \frac{1}{\rho (j, k+1)} \frac{\rho (j, k+1) \Omega (j, k+1) - \rho (j, k-1) \Omega (j, k-1) q (j, k-1)}{r (j, k+1) - r (j, k-1)}$$

and the most complicated second order difference:

$$\frac{1}{\rho} \frac{\delta}{\delta r} \left(\rho^{K_{r}} \frac{\partial q}{\partial r} \right) = \frac{\rho(j, k+1/2) K_{r}(j, k+1/2) \left[q(j, k+1) - q(j, k) \right]}{\rho(j, k) \left[r(j, k+1/2) - r(j, k-1/2) \right] \left[r(j, k+1) - r(j, k) \right]} - \frac{\rho(j, k-1/2) K_{r}(j, k-1/2) \left[q(j, k) - q(j, k-1) \right]}{\rho(j, k) \left[r(j, k+1/2) - r(j, k-1/2) \right] \left[r(j, k) - r(j, k-1) \right]}$$

Where k is finite difference grid point in the vertical and j the finite difference grid point in the horizontal.

Substituting centered finite differences in equation (13a) and collecting all terms involving q^{n+1} on the left, an algebraic equation of the form

(14a) $-A(j,k) \cdot q^{n+1} (j,k+1) + B(j,k) q^{n+1} (j,k) - C(j,k) q^{n+1} (j,k-1) = D^n (j,k)$

Where A, B, C and D are known coefficients which may be functions of time involving the physical parameters of the model and/or concentrations determined from the previous time step. Exact definition of these coefficients is given in the Appendix.

It will be noted that knowledge of q^{n+1} (j, k+1) and q^{n+1} (j, k-1) is required before one can solve (14a) for q^{n+1} (j, k). However, a knowledge of the boundary

conditions yields sufficient information to solve (14a) recursively, proceeding up from the lower boundary and down from the upper boundary. For this purpose it is convenient to rewrite (14a) using a normalized form of Gaussian elimination so that

$$q^{n+1}(j,k) = E(j,k) \cdot q(j,k+1) + F(j,k)$$

where

(15a)
$$E(j,k) = \frac{A'(j,k)}{B'(j,k) - C'(j,k) \cdot E(j,k-1)}$$

$$F(j,k) = \frac{D(j,k) + C'(j,k) \cdot F(j,k-1)}{B'(j,k) - C'(j,k) \cdot E(j,k-1)}$$

and
$$E(j, 1) = F(j, 1) = 1$$

The first point to be computed is k = 2. The boundary condition is that the diffusive flux at the lower boundary, $\binom{K_r}{\partial r}_{k=1}$ be zero. This can be satisfied by setting $\frac{\partial q}{\partial r}$ at the boundary equal to zero. For this case eqn (14a) reads

$$-A(j, 2) q^{n+1}(j, 3) + (B(j, 2) - C(j, 2)) q^{n+1}(j, 2) = D^{n}(j, 2)$$

If we now define

$$B'(j, 2) = B(j, 2) - C(j, 2)$$

$$C'(j, 2) = 0$$

The lower boundary condition is satisfied. Similarly when computing for next to the last point (in our case the last point is k = 21) we have

$$\left[B (j, 20) - A (j, 20) \right] q^{n+1} (j, 20) - C (j, 20) q^{n+1} (j, 19) = D^{n} (j, 20)$$

For this case we define

$$B'(j, 20) = B(j, 20) - A(j, 20)$$

$$A'(j, 20) = 0$$

For all other k, A= A', B= B', and C= C'

An exactly similar procedure holds for equation (13b). The above sketched procedure has been programmed for the CDC 1604. The k points run from 1 to 21 and $\Delta r = 2 \times 10^5$ cm. The j points run from 1 to 23. Point 1 is the South Pole, point 23 is the North Pole. The grid interval from j = 1 to j = 2 is

$$r_0 \Delta \mu = r_0 \Delta (\sin \phi) \int_{j=1}^{j=2} .03 r_0; r_0 \Delta (\sin \phi) \int_{j=2}^{3} .07 r_0;$$

and then $r_0 \Delta (\sin \phi) \int_{j}^{j+1} = .10 r_0$ until j = 21; at that point $r_0 \Delta (\sin \phi) \int_{j=21}^{22} .07 r_0$

and $r_0 \triangle (\sin \phi) \Big]_{j=22}^{j=23} = .03 \, r_0$. The time step used for the calculations is one week. Output is arranged so that a print out of q and ρ q for the Northern and Southern Hemisphere occurs at eight week intervals. To check continuity, the total inventory $2\pi \iint (\rho q) \, dr \, r^2 \, d\mu$ is calculated separately for the stratosphere (assumed to lie above k=10, i.e. 18 km) and the troposphere (assumed to lie below k=10). Removal mechanisms (deposition, rainout, absorption etc.) are also integrated over time for fixed latitude belts when appropriate.

Two versions of the model have been run to date:

Model 1:

$$w = v = \Omega = 0$$

 $K_r = 2x10^3 \text{ cm}^2/\text{ sec}$
 $K_{\mu} = 10^9 \text{ cm}^2/\text{sec}$
Initial conditions: $q = q_1$ at $r = 22km$, $\phi = 0^\circ$ to 5.7°N
 $q = q_2$ at $r = 24$ km, $\phi = 0^\circ$ to 5.7°N

q = 0 everywhere else

Model 2:

$$w = v = \Omega = 0$$
 $K_r = 2x10^3$ in stratosphere
 $K_r = 10^4$ in troposphere
 $K_{\mu} = 10^9$

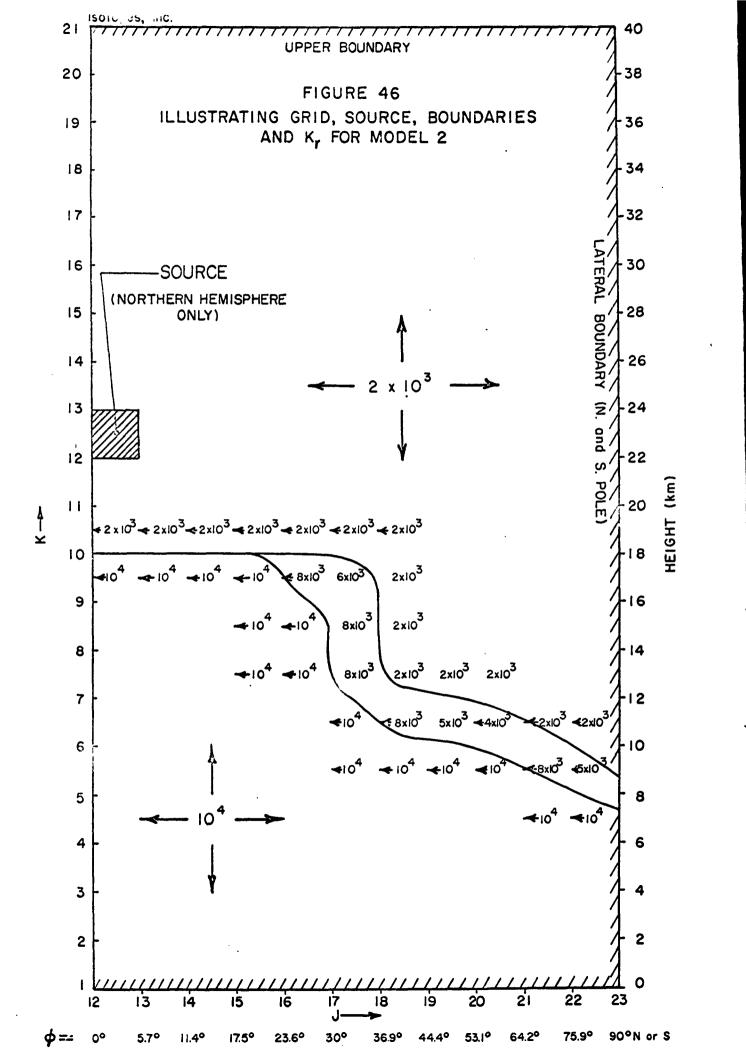
Initial conditions: Same as in Model 1.

Exact values of K_r at grid points are shown in Figure 46.

Ready to be run in next two weeks are:

Model 3:

w = v =
$$\Omega$$
 = 0
 K_r = 2×10^3 in tropical stratosphere
 K_r = 10^4 in lower polar stratosphere
 K_r = 2×10^3 in higher polar stratosphere
 K_r = 10^4 in troposphere
 K_n = 10^9



Model 4:

$$w = v = \Omega = 0$$
 $K_r = 2x10^3$ in lower tropical stratosphere

 $K_r = 8x10^2$ in upper tropical stratosphere

 $K_r = 10^4$ in lower polar stratosphere

 $K_r = 10^3$ in upper polar stratosphere

 $K_r = 10^4$ in troposphere

The primary reason for running these models is to check out the program and to gain some feeling for how the stratospheric patterns of concentration vary as the vertical diffusion coefficients are changed. It is proposed then to investigate the effect of various types of tropospheric sink mechanisms on both the ground deposition and the stratospheric patterns and inventory.

A discussion of the results of Models 1 - 4 will be reserved for a subsequent report.

SUMMARY

Work on the Star Dust model of atmospheric mixing and transfer has passed the stage of initial mathematical studies and has entered the stage of computer testing and modification of parameters. Two very preliminary models have been tested on a CDC 1604 computer and have been checked against HASP tungsten-185 data. They have shown that the observed spread of the tungsten-185 clouds can be fairly well duplicated even by the simplified model based only on eddy diffusion. More complicated models are now being prepared and tested.

The total burden of strontium-90 in the stratosphere of the Northern

Hemisphere during January to April 1962 was approximately 1.6 or 1.7

megacuries. About 1.3 or 1.4 megacuries of this had been injected by the 1961

Soviet weapon tests, and about 0.3 megacurie was the residue of debris from

tests performed before 1961. Since the stratosphere of the Southern Hemisphere

probably also contained about 0.3 megacurie of old strontium-90, the total

stratospheric burden at this time was close to 2 megacuries.

Strontium-90 produced by the 1961 Soviet tests was distinguished from strontium-90 produced by tests performed before 1959 by means of the ${\rm Sr}^{89}/{\rm Sr}^{90}$ ratio in the debris. The results of this calculation indicated that

the highest concentrations of the old, pre-1961 strontium-90 in the northern polar stratosphere were present at about 60,000 feet during January - April 1962. If this were true, the layer of air containing these high concentrations must have subsided from 70,000 feet or above, where it was located in mid-1961, to 60,000 feet during the winter of 1961 - 1962. This suggests the existence of a seasonally dependent organized transfer of some stratospheric air.

The highest concentrations of debris from the 1961 Soviet tests were found at about 50,000 feet in the northern polar stratosphere throughout the first half of 1962. Concentrations in the layers of stratospheric air below 50,000 feet were also high, but they decreased somewhat during the spring months of 1962 as a result of fallout. Concentrations at 65,000 and 70,000 feet in this region were quite low in January and February 1962, but they rose during subsequent months as a result of vertical mixing with more contaminated layers. Some of the debris entering the 65,000 to 70,000 foot layer was coming from a region not sampled by Star Dust aircraft, presumably from the layers of air above 70,000 feet, for there was a very significant increase in antimony-124 concentration in Star Dust samples during March to June 1962. Meanwhile lateral mixing was not sufficiently rapid to carry as much as 10 percent of the 1961 Soviet debris south of 30 °N and into the tropical stratosphere. Doubtless this resulted from the generally slow rate of lateral mixing within the tropical stratosphere. Evidently this rate does fluctuate, at least occasionally reaching relatively high values, for on a few Star Dust missions young Soviet debris was

encountered deep in the tropical stratosphere (for example on 29 November 1961) or young United States debris from an equatorial injection was encountered within the southern margin of the northern polar stratosphere (for example on 8 May 1962).

Results from analyses of the natural tracer isotopes, beryllium-7, lead-210 and polonium-210 and of the artificial tracer rhodium-102 have not led to any new conclusions during the past quarter. Preliminary measurements of the neutron activation products manganese-54, iron-55, cobalt-57, cobalt-58 and antimony-124 have given interesting results and a systematic analysis of these nuclides in Star Dust samples is underway.

The mean altitude of stabilization of debris from the 1961 Soviet tests was probably at least 10,000 feet above that of debris from the October 1958 Soviet tests. As a result the debris from the 1961 tests will exhibit a longer stratospheric residence time, with about half of it being detained in the stratosphere for more than a year before falling out. Thus it is not at all unlikely that the fallout rate in the spring of 1963, reinforced by debris from both the 1961 and 1962 tests, will exceed both the rates of the spring of 1959 and of the spring of 1962.

Changes which occurred in concentrations of debris near the tropopause during early 1962 suggest that the rising of the polar tropopause during the spring months is a major factor in producing a spring increase in fallout rate

at the ground, especially in years (such as 1959 and 1962) which follow major Soviet fall test series. Evidently horizontal mixing through the tropopause gap region (or through the sloping polar tropopause) is also important in contributing to the spring increase. Its relative importance is greatest, no doubt, during years (such as 1960 and 1961) when concentrations of debris in the lowest layers of the polar stratosphere are low compared to those at 50,000 and 55,000 feet.

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APPENDIX

For clarity the coefficients A, B, C, D appearing in equation (14a) are indicated by AV, BV, CV, DV; the coefficients appearing in the analogue of (2b) are indicated by AH, BH, CH, DH. The letter j is the finite difference grid point in the poleward direction; the letter k is the finite difference grid point in the vertical direction. The equations below are written for a constant grid interval, r_0 $\Delta\mu$ and Δr in the horizontal and vertical directions respectively. These equations must be modified for the unequal grid interval j = 1, 2; j = 2, 3; j = 21, 22; 22, 23. For convenience, K_r $(j, k+1/2) \equiv K$ $(j, k); K_r$ $(j, k-1/2) \equiv K$ (j, k-1/2) and similarly for the "j" dependence. Also

$$V = v \cos \phi$$

$$(1-\mu^{2}) K_{\mu} = K'$$

$$K_{r} = K$$

$$\frac{\rho(j, k+1/2)}{\rho_{k}} = \frac{\rho(j, k+1) + \rho(j, k)}{2 \rho(j, k)} = \frac{\overline{\rho}}{2} (j, k)$$

$$\frac{\rho(j, k-1/2)}{\rho_{k}} = \frac{\rho(j, k) + \rho(j, k-1)}{2 \rho(j, k)} = \frac{\overline{s}}{2} (j, k)$$

With these definitions:

$$AV(j,k) = \left(\frac{\Delta t}{4\Delta r}\right) \cdot \left[\frac{\overline{\rho}(j,k) \cdot K(j,k)}{\Delta r} - \frac{\rho(j,k+1) \Omega (j,k+1) - w(j,k)}{\rho(j,k)}\right]$$

BV (j, k) =
$$1 + \frac{\Delta t}{4(\Delta r)^2} \left[\overline{\rho} (j, k) \cdot K (j, k) + \overline{s} \cdot K (j, k-1) \right]$$

$$CV(j,k) = \left(\frac{\Delta t}{4 \Delta r}\right) \cdot \left[\frac{\overline{s}(k,j) \cdot K(j,k-1)}{\Delta r} + \frac{\rho(j,k-1)\Omega}{\rho(j,k)}\Omega(j,k-1) + w(j,k)\right]$$

$$AH(j,k) = \frac{\Delta t}{2 r_0 \Delta \mu} \left[\frac{K'(j,k)}{r_0 \Delta \mu} - \frac{V(j,k)}{2}\right]$$

BH (j, k) =
$$1 + \frac{\Delta_{2t}}{2r_{0}^{2}(\Delta_{\mu})^{2}} \left[K'(j, k) + K'(j-1, k)\right]$$

CH (j, k) =
$$\frac{\Delta t}{2 r_0 (\Delta \mu)}$$
 $\frac{K'(j-1, k)}{r_0 \Delta \mu}$ + $\frac{V(j, k)}{2}$

GV (j, k) = 1 -
$$\frac{\Delta t}{4 (\Delta r)^2} \left[\overline{\rho} \cdot K(j, k) + \overline{s} \cdot K(j, k-1) \right]$$
 -

$$\frac{\Delta t}{(r_0 \Delta \mu)^2} \left[K'(j,k) + K'(j-1,k) \right]$$

GH (j, k) = 1 -
$$\frac{\Delta t}{2 \left[r_0 \Delta \mu \right]} 2 \left[K'(j,k) + K'(j-1,k) \right]$$

$$\frac{\Delta t}{2(|\Lambda|r)^{2i}} \left[\rho^{i} K (j,k) - \overline{s} K (j,k-1) \right]$$

DV
$$(j, K) = AV(j, k) \cdot q^{n}(j, k+1) + GV(j, k) \cdot q^{n}(j, k) + CV(j, k) q^{n}(j, k-1) + 2AH(j, k) \cdot q^{n}(j+1, k) + 2CH \cdot q^{n}(j-1, k)$$

DH
$$(j,k)$$
 AH $(j,k) \cdot q^n (j+1,k) + GH $(j,k) \cdot q^n (j,k) + CH(j,k) q^n (j-1,k) + 2 \text{ AV } (j,k) \cdot q^n (j,k+1) + 2 \text{ CV } (j,k) \cdot q^n (j,k-1)$$